Supplementary Material for "Finite-Time Sliding-Mode Control for Semi-Markov Systems With Delayed Impulses"

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In system (1), considering the semi-Markov chain r be $K=1,2$ with generator $\Pi=[-1.5,1.5;0.8,-0.8]$.

For mode 1:

$$A_1=\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}, B_1=\begin{bmatrix}1 & 1.2\\ 0.2 & 2\end{bmatrix}, D_1=\begin{bmatrix}2 & 1.5\\ 1 & 1.1\end{bmatrix}.$$

For mode 2:

$$A_2=I, B_2=\begin{bmatrix}3 & 2.2\\ 2 & 2.5\end{bmatrix}, D_2=\begin{bmatrix}0.3 & -1.5\\ -2.1 & 0.8\end{bmatrix}.$$

The $C=I$, and the nonlinear function $h(\cdot)=\tanh(\cdot)$, the delay is $\upsilon(t)=e^{-t}$. The system (1) is subject to impulses as follows: $\xi(t_k)=Y_1\left(\xi\left(t_k^-\right)\right)+Y_2\left(\xi\left(\left(t_k-\theta_k\right)^-\right)\right)$, where $Y_1=\begin{pmatrix}0.6 & -0.1\\ -0.1 & 0.6\end{pmatrix}, Y_2=\begin{pmatrix}0.7 & 0.2\\ 0.1 & 0.8\end{pmatrix}$. The parameters of the sliding mode function (2) and SMC law (3) are given: $R_1=R_2=I, Q=I/2, W=2I, H=4I, \zeta=2, \alpha=\beta=3/5$. Let $V=\xi^{\dagger}(t)P(t)R_r\xi(t)$, $P(t)=\left(0.1e^{-5t}+1\right)$ from $\xi(t_k)$, one can get $\lambda_1=0.992$, $\lambda_2=1.2128$. Let $N=3, \vartheta=0.6$, due to Theorem 1 with (5) and Theorem 2, the impulse sequence satisfies $\{t_1=0.15, t_2=0.3, t_3=0.45, t_4=0.6, t_5=0.75, t_6=0.9\}$ and the impulsive delay is $\theta_1=0.1, \theta_2=0.08, \theta_3=0.05, \theta_4=0.12, \theta_5=0.04, \theta_6=0.1$. Let $\sigma=4$, then the (15) is satisfied.

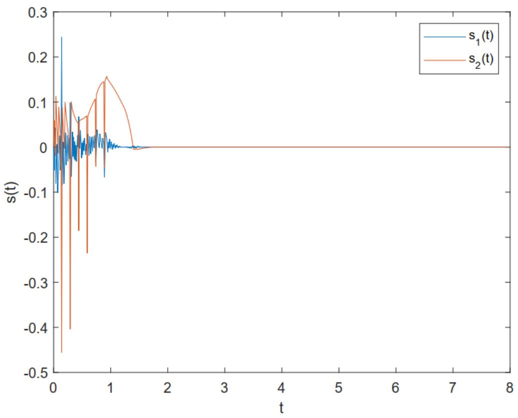

Fig.S1. The finite-time stabilization trajectory of the sliding mode dynamics.

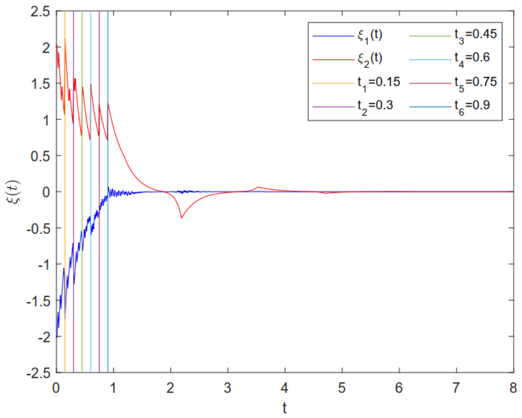

Fig.S2. The finite-time stabilization trajectory of the system (1) with $\zeta = 0.5, W = 0.5I$, $\alpha = \beta = 13/15$.

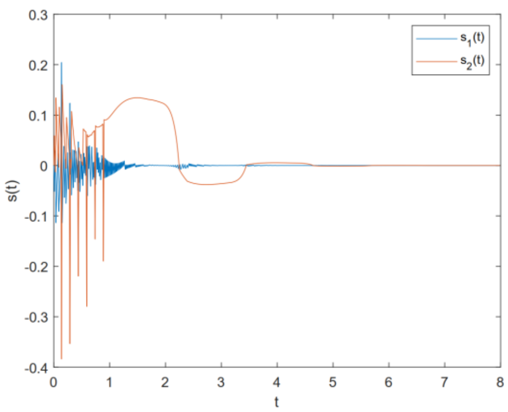

Fig.S3. The finite-time stabilization trajectory of the sliding mode dynamics.

Remark 1: In order to eliminate the system delay $\upsilon(t)$, this letter mainly introduces system delay terms $\tilde{\mu}(t) = -sign(s(t))\left(\delta\|\phi(t)\| + \eta\|\phi(t-\upsilon(t))\|\right)$ and $QMD_r h(\phi(t-\upsilon(t)))$ in the sliding mode controller (3) and the sliding mode manifold (2). In this way, the term with system delay can be offset in (7) to achieve the purpose.

Remark 2: Here, θ_k can represent both time-varying/random variable and fixed impulse delay.

No matter which impulse delay it is, as long as it satisfies the setting of this letter, its upper bound is known, and the time delay θ_k can be scaled to a constant in the theoretical derivation. Specifically, the sequence (5) related to the impulse delay is established using the set impulse constraint condition (4). Then, the delay is iterated to solve the impulse delay problem in the reachable stage. Furthermore, combined with the maximum-minimum impulse interval, the impulse delay is transformed to solve the delay problem in the sliding stage of the system.

Remark 3: This letter uses $\tilde{\xi}(t)=\sum_{i=1}^{N_2(t)}\left(\xi\left(t_i^-\right)-g\left(\xi\left(t_i^-\right),\xi\left(t_i-\theta_i\right)^-\right)\right)+\xi(t)-\xi(0)$ to replace the piecewise linear function in [8], constructs a novel sliding mode control strategy, simplifies the derivative of the sliding surface, and makes the analysis process more accessible.
This letter fully considers the influence of impulse degree with or without delay on the convergence time. Many studies only discuss the role of impulse without delay [10] and the influence of impulse with delay [5]. A few papers also point out that exponential stability is achieved when affected by both [6]. However, this letter discusses the problem of the simultaneous existence of two types of impulses. It establishes a new impulse sequence to achieve finite time convergence of the system, classifies and discusses the influence of impulse degree on stability under different value ranges, and gives the convergence time estimation formula. The results show that the settling time is related to the initial state, various impulse time series and impulse degree. It is worth noting that the delay impulse degree $\lambda_2\in(-\infty,+\infty)$, when $\lambda_2=0$, the system (1) degenerates to the impulse without delay. The integral sliding mode control strategy and convergence time estimation formula of this letter are still valid, indicating that it has strong flexibility.

Remark 4: When considering the semi-Markov chain rule, according to the sufficient condition (15) for system stability in Theorem 2, the various control gains selected are required to satisfy (15) to ensure that the system (1) reaches stability under the action of the sliding mode controller. Otherwise, the convergence and divergence cannot be determined. Secondly, according to the estimation formula of the finite time convergence time, it can be found that in addition to the pulse degree with or without time delay affecting the settling time T, the control gains ζ and W are also essential parameters affecting the settling time T. It is worth mentioning that when the system reaches the vicinity of the sliding surface, ζ and β the power accelerate the system to reach the sliding surface. When the system slides on the sliding surface to the vicinity of the origin, W and α the power accelerate the system to reach the origin. Fig. S1 shows the trajectory of the finite-time stable sliding mode dynamics.

We give the comparison chart in the simulation Fig. S2 and Fig. S3. When ζ and W are smaller, α and β are closer to 1, the long tail effect of system convergence is more apparent, and the convergence time is longer.