

Supplementary File for “A Decision Variables Classification-Based Evolutionary Algorithm for Constrained Multi-Objective Optimization Problems”

I. SUPPLEMENTARY RESULTS

TABLE S-1
MEAN IGD^+ RESULTS OBTAINED BY THE ALGORITHM AT DIFFERENT CLASSIFICATION ACCURACIES

Problem	Classification Accuracy				
	0.2	0.4	0.6	0.8	1.0
SDC5	1.4170e+3 (2.30e+2) -	1.1783e+3 (2.35e+2) =	1.2083e+3 (1.80e+2) =	1.2738e+3 (2.34e+2) =	1.0948e+3 (1.88e+2)
SDC7	3.6912e+0 (3.95e-1) -	3.3972e+0 (6.18e-1) -	3.5983e+0 (3.44e-1) -	2.9261e+0 (4.45e-1) -	1.9154e+0 (4.29e-1)
SDC8	7.7933e+1 (2.46e+1) -	7.0432e+1 (2.14e+1) -	4.5916e+1 (1.94e+1) -	2.7592e+1 (2.62e+1) -	6.4932e+0 (7.50e+0)
SDC11	6.5654e+2 (1.61e+2) -	6.3262e+2 (1.84e+2) -	5.5333e+2 (1.55e+2) -	5.2858e+2 (8.97e+1) -	3.8853e+2 (7.72e+1)
SDC12	1.2897e+2 (3.32e+1) -	9.4284e+1 (1.87e+1) -	6.8202e+1 (1.98e+1) -	5.4774e+1 (2.69e+1) -	1.2419e+1 (8.93e+0)
+/-=	0/5/0	0/4/1	0/4/1	0/4/1	

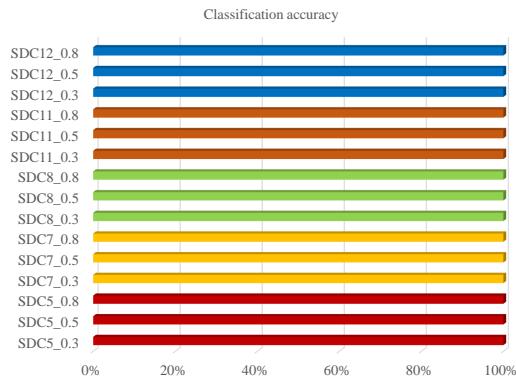


Fig. S-1. The classification accuracy of DVCEA on the different variant problems. The leftmost column shows the different variant functions, and the bar chart on the right shows the classification accuracy of DVCEA on each problem.

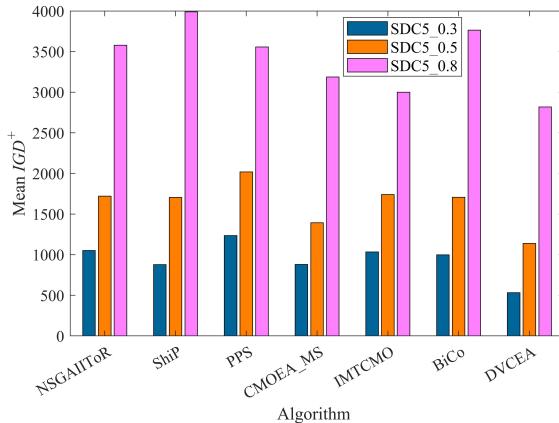


Fig. S-2. The average IGD^+ diagram of the algorithms on the SDC5 variant problem.

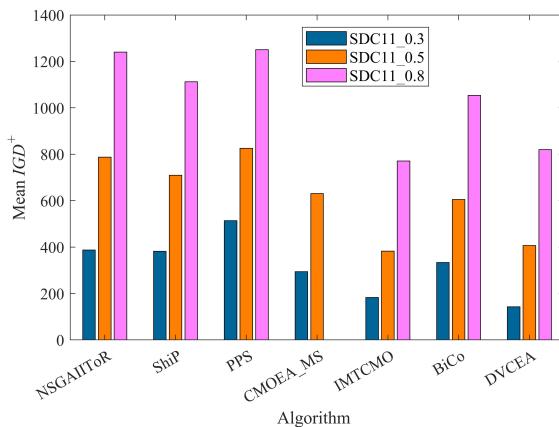


Fig. S-3. The average IGD^+ diagram of the algorithms on the SDC11 variant problem.

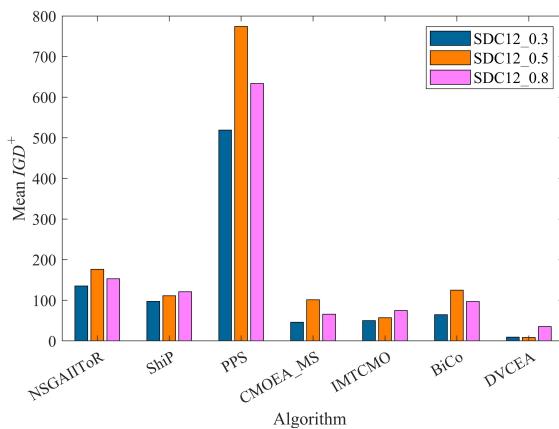


Fig. S-4. The average IGD^+ diagram of the algorithms on the SDC12 variant problem.

TABLE S-2
MEAN IGD^+ RESULTS OF DVCEA AND COMPARED ALGORITHM ON THE DIFFERENT VARIANT PROBLEMS

Problem	The method based on single population and single stage		The method based on two-stage		The method based on two-population		The proposed algorithm
	NSGAIIToR	ShiP	PPS	CMOEA_MS	IMTCMO	BiCo	DVCEA
SDC5_0.3	1.0506e+3 (2.20e+2) -	8.7818e+2 (1.13e+2) -	1.2358e+3 (2.78e+2) -	8.8012e+2 (1.12e+2) -	1.0330e+3 (1.47e+2) -	9.9805e+2 (2.53e+2) -	5.3272e+2 (1.34e+2)
SDC5_0.5	1.7197e+3 (2.32e+2) -	1.7046e+3 (1.32e+2) -	2.0192e+3 (3.90e+2) -	1.3921e+3 (1.79e+2) -	1.7398e+3 (2.75e+2) -	1.7076e+3 (2.39e+2) -	1.1389e+3 (2.05e+2)
SDC5_0.8	3.5787e+3 (4.43e+2) -	3.9895e+3 (6.09e+2) -	3.5576e+3 (8.23e+2) -	3.1866e+3 (4.16e+2) =	2.9997e+3 (5.15e+2) =	3.7638e+3 (5.07e+2) -	2.8191e+3 (3.06e+2)
SDC7_0.3	5.3365e+0 (6.44e-1) -	NaN (72.6%) -	1.0065e+1 (4.26e-1) -	8.0743e+0 (1.54e+0) -	3.7679e+0 (5.62e-1) -	1.0278e+1 (6.92e-1) -	1.7892e+0 (4.45e-1)
SDC7_0.5	6.6283e+0 (7.73e-1) -	NaN (0.7%) -	1.1807e+1 (5.45e-1) -	1.0548e+1 (1.93e+0) -	4.2258e+0 (1.01e+0) -	1.2400e+1 (5.37e-1) -	2.1953e+0 (2.83e-1)
SDC7_0.8	6.3424e+0 (1.25e+0) -	NaN (0.5%) -	1.3779e+1 (3.33e-1) -	1.2066e+1 (2.01e+0) -	5.1587e+0 (1.04e+0) -	1.4703e+1 (5.74e-1) -	3.5961e+0 (3.73e-1)
SDC8_0.3	1.2345e+2 (5.94e+1) -	2.5768e+1 (2.15e+1) =	3.2194e+2 (6.40e+1) -	1.7041e+1 (1.02e+1) =	1.6814e+1 (1.95e+1) =	1.0198e+2 (4.93e+1) -	1.5413e+1 (1.80e+1)
SDC8_0.5	1.7979e+2 (7.97e+1) -	3.8513e+1 (1.37e+1) -	5.3891e+2 (9.83e+1) -	2.4159e+1 (1.23e+1) -	3.5584e+1 (2.45e+1) -	2.2718e+2 (5.72e+1) -	7.8429e+0 (8.68e+0)
SDC8_0.8	2.7172e+2 (1.57e+2) -	NaN (62.9%) -	6.4694e+2 (1.65e+2) -	2.2542e+1 (6.44e+0) =	4.6931e+1 (3.34e+1) =	3.2361e+2 (8.54e+1) -	3.1742e+1 (1.77e+1)
SDC11_0.3	3.8756e+2 (9.31e+1) -	3.8174e+2 (7.58e+1) -	5.1351e+2 (9.47e+1) -	2.9374e+2 (7.95e+1) -	1.8311e+2 (4.03e+1) -	3.3364e+2 (1.20e+2) -	1.4293e+2 (5.55e+1)
SDC11_0.5	7.8762e+2 (1.48e+2) -	7.0951e+2 (1.45e+2) -	8.2519e+2 (1.39e+2) -	6.3029e+2 (1.33e+2) -	3.8264e+2 (5.12e+1) =	6.0465e+2 (1.18e+2) -	4.0670e+2 (8.43e+1)
SDC11_0.8	1.2401e+3 (1.09e+2) -	1.1122e+3 (1.21e+2) -	1.2506e+3 (1.33e+2) -	NaN (82.6%) -	7.7080e+2 (1.34e+2) =	1.0532e+3 (1.57e+2) -	8.1955e+2 (1.49e+2)
SDC12_0.3	1.3532e+2 (4.40e+1) -	9.7383e+1 (3.61e+1) -	5.1906e+2 (6.68e+1) -	4.5950e+1 (1.64e+1) -	4.9840e+1 (1.12e+1) -	6.4374e+1 (3.20e+1) -	9.3665e+0 (8.45e+0)
SDC12_0.5	1.7621e+2 (4.80e+1) -	1.1114e+2 (4.23e+1) -	7.7425e+2 (8.89e+1) -	1.0131e+2 (3.99e+1) -	5.6977e+1 (2.76e+1) -	1.2497e+2 (1.92e+1) -	8.3027e+0 (7.46e+0)
SDC12_0.8	1.5294e+2 (4.93e+1) -	1.2106e+2 (3.09e+1) -	6.3344e+2 (1.22e+2) -	6.5794e+1 (2.27e+1) -	7.4977e+1 (2.25e+1) -	9.6838e+1 (3.35e+1) -	3.5400e+1 (1.69e+1)
+/-=	0/15/0	0/14/1	0/15/0	0/12/3	0/9/6	0/15/0	

TABLE S-3
MEAN HV RESULTS OF DVCEA AND COMPARED ALGORITHM ON THE DIFFERENT VARIANT PROBLEMS

Problem	The method based on single population and single stage		The method based on two-stage		The method based on two-population		The proposed algorithm
	NSGAIIToR	ShiP	PPS	CMOEA_MS	IMTCMO	BiCo	DVCEA
SDC5_0.3	9.3937e-01 (1.18e-01) -	9.7148e-01 (7.51e-02) -	1.0697e+00 (5.46e-02) -	9.8832e-01 (7.69e-02) -	1.1189e+00 (2.58e-02) -	1.1048e+00 (5.68e-02) -	1.1780e+00 (1.29e-02)
SDC5_0.5	1.1699e+00 (8.58e-02) -	1.1067e+00 (1.51e-01) -	1.2061e+00 (1.78e-03) -	1.1002e+00 (1.40e-01) =	1.2079e+00 (6.03e-04) =	1.2081e+00 (4.91e-04) =	1.2083e+00 (6.35e-04)
SDC5_0.8	9.7213e-01 (1.21e-01) -	8.3537e-01 (2.03e-01) -	1.0170e+00 (8.60e-02) -	1.0375e+00 (1.05e-01) =	1.0772e+00 (4.35e-02) =	1.0045e+00 (5.35e-02) -	1.0938e+00 (2.53e-02)
SDC7_0.3	1.0771e+00 (2.56e-02) -	9.8657e-01 (9.45e-02) -	7.9563e-01 (2.91e-02) -	8.4277e-01 (2.14e-01) -	1.1314e+00 (1.82e-02) -	7.9929e-01 (5.01e-02) -	1.1721e+00 (9.39e-03)
SDC7_0.5	1.0558e+00 (3.04e-02) -	NaN (0.00e+00) -	7.8345e-01 (3.77e-02) -	7.2415e-01 (2.02e-01) -	1.1287e+00 (3.03e-02) -	7.4694e-01 (3.70e-02) -	1.1717e+00 (5.15e-03)
SDC7_0.8	1.0527e+00 (7.25e-02) -	NaN (0.00e+00) -	6.9783e-01 (2.30e-02) -	7.1770e-01 (1.87e-01) -	1.1001e+00 (2.80e-02) -	6.3723e-01 (3.73e-02) -	1.1477e+00 (6.70e-03)
SDC8_0.3	1.2043e+00 (8.44e-03) -	1.2098e+00 (1.15e-04) =	1.1492e+00 (1.17e-02) -	1.2079e+00 (1.66e-03) -	1.2100e+00 (7.27e-05) =	1.2074e+00 (1.61e-03) -	1.2099e+00 (1.16e-04)
SDC8_0.5	1.2054e+00 (2.28e-03) -	1.2093e+00 (2.44e-04) -	1.1215e+00 (3.19e-02) -	1.2041e+00 (3.30e-03) -	1.2098e+00 (1.78e-04) =	1.2050e+00 (1.40e-03) -	1.2099e+00 (7.67e-05)
SDC8_0.8	1.1895e+00 (1.60e-02) -	1.2086e+00 (1.04e-03) -	1.0971e+00 (2.42e-02) -	1.2037e+00 (2.32e-03) -	1.2097e+00 (3.24e-04) =	1.1983e+00 (4.60e-03) -	1.2097e+00 (1.36e-04)
SDC11_0.3	1.1315e+00 (3.54e-02) -	1.0899e+00 (1.04e-01) -	1.0731e+00 (4.84e-02) -	1.0955e+00 (9.32e-02) -	1.1911e+00 (8.75e-03) =	1.1476e+00 (5.35e-02) -	1.1964e+00 (1.07e-02)
SDC11_0.5	1.1965e+00 (4.81e-03) -	1.1986e+00 (5.22e-03) -	1.1945e+00 (4.84e-03) -	1.1994e+00 (6.19e-03) -	1.2065e+00 (9.41e-04) =	1.2019e+00 (3.08e-03) -	1.2054e+00 (1.99e-03)
SDC11_0.8	1.1235e+00 (1.54e-02) -	1.1404e+00 (1.34e-02) -	1.1201e+00 (1.94e-02) -	1.1441e+00 (1.91e-02) -	1.1743e+00 (1.18e-02) =	1.1466e+00 (1.83e-02) -	1.1699e+00 (1.39e-02)
SDC12_0.3	1.2033e+00 (6.68e-03) -	1.2074e+00 (1.52e-03) -	1.1798e+00 (1.54e-02) -	1.2017e+00 (3.95e-03) -	1.2098e+00 (1.13e-04) -	1.2083e+00 (6.21e-04) -	1.2099e+00 (5.52e-05)
SDC12_0.5	1.1945e+00 (1.96e-02) -	1.2023e+00 (3.55e-03) -	1.0968e+00 (2.42e-02) -	1.1795e+00 (2.28e-02) -	1.2093e+00 (5.47e-04) -	1.2030e+00 (2.87e-03) -	1.2099e+00 (1.09e-04)
SDC12_0.8	1.1999e+00 (7.43e-03) -	1.2029e+00 (1.60e-03) -	1.0766e+00 (1.83e-02) -	1.1872e+00 (1.01e-02) -	1.2090e+00 (5.35e-04) -	1.2036e+00 (3.63e-03) -	1.2094e+00 (3.80e-04)
+/-=	0/15/0	0/14/1	0/15/0	0/13/2	0/7/8	0/14/1	

TABLE S-4
MEAN HV RESULTS OF DVCEA AND COMPARED ALGORITHMS ON SDC AND ZXH_CF PROBLEMS

Problem	The method based on single population and single stage		The method based on two-stage		The method based on two-population		The proposed algorithm	
	NSGAII>ToR	ShiP	PPS	CMOEAs_MS	IMTCMO	BiCo	DVCEA	
SDC1	1.1256e+00 (2.13e-02) =	1.1014e+00 (8.64e-02) =	1.0003e+00 (6.00e-02) -	1.1323e+00 (1.42e-02) +	1.1067e+00 (3.04e-02) -	1.1160e+00 (3.22e-02) =	1.1287e+00 (3.67e-03)	
SDC2	1.2098e+00 (3.92e-04) =	1.2099e+00 (2.01e-04) =	1.2088e+00 (2.44e-04) -	1.2095e+00 (1.14e-02) -	1.2100e+00 (4.03e-08) -	1.2098e+00 (3.42e-04) =	1.2100e+00 (1.89e-07)	
SDC3	1.0336e+00 (1.03e-02) =	NaN (0%) -	9.1307e-01 (3.59e-02) -	9.9866e-01 (1.40e-02) -	9.9856e-01 (2.75e-02) -	9.3073e-01 (4.29e-02) -	1.0279e+00 (1.22e-02)	
SDC4	NaN (80%) -	NaN (0%) -	1.2099e+00 (1.01e-04) -	9.8220e-01 (1.81e-01) -	1.2100e+00 (1.66e-06) =	1.0930e+00 (8.74e-02) -	1.2100e+00 (8.24e-07)	
SDC5	1.1699e+00 (8.58e-02) -	1.1067e+00 (1.51e-01) -	1.2061e+00 (1.78e-03) -	1.1002e+00 (1.40e-01) =	1.2079e+00 (6.03e-04) -	1.2081e+00 (4.91e-04) =	1.2083e+00 (6.35e-04)	
SDC6	NaN (0%) -	NaN (0.7%) -	1.2026e+00 (5.89e-03) -	1.1827e+00 (2.42e-02) -	1.2089e+00 (5.58e-04) =	1.2062e+00 (3.70e-03) =	1.2085e+00 (2.59e-03)	
SDC7	1.0558e+00 (3.04e-02) -	NaN (0%) -	7.8345e-01 (3.77e-02) -	7.2415e-01 (2.02e-01) -	1.1287e+00 (3.03e-02) -	7.4694e-01 (3.70e-02) -	1.1717e+00 (5.15e-03)	
SDC8	1.2054e+00 (2.28e-03) -	1.2093e+00 (2.44e-04) -	1.1215e+00 (3.19e-02) -	1.2041e+00 (3.30e-03) -	1.2098e+00 (1.78e-04) =	1.2050e+00 (1.40e-03) -	1.2099e+00 (7.67e-05)	
SDC9	1.1378e+00 (5.47e-02) -	NaN (0%) -	1.0856e+00 (3.84e-02) -	NaN (80%) -	9.7973e-01 (1.03e-01) -	1.1192e+00 (1.68e-02) -	1.1965e+00 (9.91e-03)	
SDC10	9.4602e-01 (3.77e-02) =	6.2012e-01 (5.22e-02) -	8.8020e-01 (3.68e-02) -	9.2874e-01 (9.58e-02) =	9.1989e-01 (7.44e-02) =	9.2376e-01 (6.72e-02) =	9.4535e-01 (2.67e-02)	
SDC11	1.1965e+00 (4.81e-03) -	1.1986e+00 (5.22e-03) -	1.1945e+00 (4.84e-03) -	1.1994e+00 (6.19e-03) -	1.2065e+00 (9.41e-04) =	1.2019e+00 (3.08e-03) -	1.2054e+00 (1.99e-03)	
SDC12	1.1945e+00 (1.96e-02) -	1.2023e+00 (3.55e-03) -	1.0968e+00 (2.42e-02) -	1.1795e+00 (2.28e-02) -	1.2093e+00 (5.47e-04) -	1.2030e+00 (2.87e-03) -	1.2099e+00 (1.09e-04)	
SDC13	7.4988e-01 (1.09e-01) -	NaN (0%) -	8.8486e-01 (2.86e-02) -	NaN (60%) -	8.5309e-01 (1.00e-01) -	8.4018e-01 (1.42e-01) -	9.8911e-01 (2.10e-02)	
SDC14	NaN (0%) -	NaN (0%) -	NaN (60%) -	NaN (0%) -	NaN (80%) =	NaN (20%) -	NaN (80%)	
SDC15	1.3301e+00 (5.16e-05) +	1.3301e+00 (7.85e-05) +	1.3170e+00 (1.17e-03) -	1.3301e+00 (7.66e-05) +	1.3293e+00 (5.06e-04) -	1.3301e+00 (5.12e-05) +	1.3300e+00 (5.91e-05)	
ZXH_CF1	1.3032e+00 (7.14e-04) -	1.3023e+00 (5.88e-04) -	1.2853e+00 (2.17e-03) -	1.3072e+00 (3.38e-04) -	1.3016e+00 (5.48e-04) -	1.3076e+00 (1.63e-04) =	1.3077e+00 (1.99e-04)	
ZXH_CF2	1.2287e+00 (1.65e-01) =	NaN (90%) -	1.1716e+00 (1.42e-01) -	1.2329e+00 (1.33e-01) =	1.2319e+00 (7.99e-02) -	1.2442e+00 (1.35e-01) =	1.3004e+00 (3.20e-02)	
ZXH_CF3	1.2483e+00 (3.19e-03) =	1.2226e+00 (7.81e-02) -	NaN (80%) -	1.2527e+00 (1.57e-03) +	1.2231e+00 (2.68e-03) -	1.2536e+00 (1.09e-03) +	1.2504e+00 (1.66e-03)	
ZXH_CF4	1.2119e+00 (2.50e-02) -	1.1628e+00 (1.17e-01) -	1.1957e+00 (1.61e-02) -	1.2087e+00 (2.35e-02) -	1.1934e+00 (2.08e-02) -	1.2176e+00 (1.20e-02) -	1.2240e+00 (6.85e-04)	
ZXH_CF5	9.4624e-01 (2.49e-01) -	NaN (90%) -	1.0050e+00 (1.68e-01) -	NaN (65%) -	1.0493e+00 (1.44e-01) -	8.9048e-01 (2.48e-01) -	1.2340e+00 (4.57e-02)	
ZXH_CF6	7.8418e-01 (6.35e-03) -	7.8167e-01 (2.99e-03) -	7.6781e-01 (4.93e-03) -	8.0662e-01 (4.71e-03) =	7.3923e-01 (6.74e-03) -	8.0620e-01 (2.82e-03) =	8.0481e-01 (2.51e-03)	
ZXH_CF7	1.2294e+00 (1.72e-01) =	1.1027e+00 (2.68e-01) =	1.2347e+00 (3.47e-02) -	NaN (34.7%) -	1.2558e+00 (5.34e-03) -	1.2639e+00 (2.59e-02) -	1.2814e+00 (3.25e-03)	
ZXH_CF8	1.0806e+00 (2.93e-03) =	1.0772e+00 (4.35e-03) =	9.5919e-01 (9.80e-02) -	6.8750e-01 (8.48e-02) -	9.9664e-01 (6.05e-03) -	1.0810e+00 (6.31e-03) =	1.0783e+00 (5.00e-03)	
ZXH_CF9	1.0613e+00 (1.70e-01) -	1.1652e+00 (2.65e-03) -	1.1111e+00 (7.06e-02) -	1.1703e+00 (2.71e-03) =	1.1505e+00 (6.74e-04) -	1.1702e+00 (2.79e-04) -	1.1708e+00 (4.18e-04)	
ZXH_CF10	1.0351e+00 (1.69e-01) -	1.1873e+00 (2.28e-02) -	9.0067e-01 (1.15e-01) -	9.9874e-01 (1.17e-01) -	1.1702e+00 (2.63e-02) -	1.1881e+00 (2.76e-02) -	1.1983e+00 (5.14e-04)	
ZXH_CF11	6.7277e-01 (2.44e-01) -	9.9232e-01 (2.11e-03) -	4.9649e-01 (1.69e-01) -	1.0023e+00 (1.31e-03) =	9.8332e-01 (1.74e-03) -	1.0021e+00 (1.19e-03) =	1.0017e+00 (1.62e-03)	
ZXH_CF12	1.0452e+00 (6.94e-02) -	1.2432e+00 (9.82e-02) =	1.0333e+00 (8.96e-02) -	1.2103e+00 (1.18e-01) -	1.2845e+00 (4.60e-02) =	1.2680e+00 (8.77e-02) =	1.3088e+00 (1.94e-02)	
ZXH_CF13	9.1115e-01 (1.34e-01) -	1.0572e+00 (1.79e-02) -	9.2766e-01 (1.22e-01) -	9.9197e-01 (7.84e-02) -	1.0645e+00 (7.63e-05) -	1.0594e+00 (1.73e-02) -	1.0652e+00 (3.36e-05)	
ZXH_CF14	8.4970e-01 (2.57e-02) -	8.5760e-01 (2.19e-04) -	8.4868e-01 (9.33e-04) -	8.3381e-01 (3.93e-02) -	8.5099e-01 (6.45e-04) -	8.5767e-01 (2.57e-04) -	8.5894e-01 (1.23e-04)	
ZXH_CF15	1.0159e+00 (1.11e-01) -	1.0504e+00 (1.11e-01) -	1.0156e+00 (8.51e-02) -	1.0787e+00 (1.92e-01) -	1.1213e+00 (7.11e-02) =	1.1482e+00 (2.38e-02) =	1.1532e+00 (2.14e-02)	
ZXH_CF16	1.0475e+00 (5.76e-02) -	1.0584e+00 (5.65e-02) -	1.0860e+00 (1.06e-02) -	1.0119e+00 (4.16e-02) -	1.1010e+00 (9.01e-05) -	1.1024e+00 (3.36e-05) +	1.1023e+00 (9.38e-05)	

Problem	DVCEA_5_2	DVCEA_5_6	DVCEA_5_11	DVCEA_10_2	DVCEA_10_4	DVCEA_10_6	DVCEA_10_11	DVCEA_LM	DVCEA_5_4
SDC1	48%	100%	100%	48%	100%	100%	100%	83.60%	100%
SDC2	41%	63.00%	66.10%	41%	73.80%	76.60%	78.50%	51.50%	63.60%
SDC3	46%	100%	100%	46%	100%	100%	100%	69.60%	100%
SDC4	50%	100%	100%	50%	100%	100%	100%	63.80%	100%
SDC5	47%	100%	100%	47%	100%	100%	100%	55.10%	100%
SDC6	47%	100%	100%	47%	100%	100%	100%	68.00%	100%
SDC7	50%	100%	100%	50%	100%	100%	100%	29.60%	100%
SDC8	49%	100%	100%	49%	100%	100%	100%	73.70%	100%
SDC9	46%	100%	100%	46%	100%	100%	100%	55.40%	100%
SDC10	46%	100%	100%	46%	100%	100%	100%	79.40%	100%
SDC11	43%	100%	100%	43%	100%	100%	100%	69.00%	100%
SDC12	50%	100%	100%	50%	100%	100%	100%	78.60%	100%
SDC13	49%	100%	100%	49%	100%	100%	100%	64.70%	100%
SDC14	49%	100%	100%	49%	100%	100%	100%	58.80%	100%
SDC15	46%	97%	97%	46%	97.40%	97.10%	97.10%	21.90%	97%

Fig. S-5. The heat map regarding classification accuracy of various parameter combinations on the 15 SDC test problems.

TABLE S-5
MEAN IGD^+ RESULTS FOR DIFFERENT COMBINATIONS OF PARAMETER K AND PE

Problem	DVCEA_5_2	DVCEA_5_6	DVCEA_5_11	DVCEA_10_2	DVCEA_10_4	DVCEA_10_6	DVCEA_10_11	DVCEA_5_4
	$K=5$	$K=5$	$K=5$	$K=10$	$K=10$	$K=10$	$K=10$	$K=5$
	$PE=2$	$PE=6$	$PE=11$	$PE=2$	$PE=4$	$PE=6$	$PE=11$	$PE=4$
SDC1	1.1319e+1 (1.78e+0) =	1.0227e+1 (1.25e-1) =	1.0279e+1 (2.93e-1) =	1.0601e+1 (8.58e-1) =	1.0148e+1 (4.40e-2) =	1.0216e+1 (9.30e-2) =	1.0191e+1 (1.01e-1) =	1.0148e+1 (1.41e-1)
SDC2	5.6734e-1 (3.55e-2) -	5.1776e-1 (1.01e-1) -	5.1588e-1 (6.93e-2) -	5.6747e-1 (5.07e-2) -	4.4141e-1 (1.30e-1) =	4.5413e-1 (1.49e-1) =	4.3380e-1 (1.77e-2) =	3.8472e-1 (1.43e-1)
SDC3	1.3172e+0 (4.74e-2) =	1.3501e+0 (5.28e-2) =	1.3283e+0 (5.90e-2) =	1.3104e+0 (6.48e-2) =	1.3741e+0 (9.05e-2) =	1.3493e+0 (5.08e-2) =	1.3548e+0 (3.27e-2) =	1.3433e+0 (6.78e-2)
SDC4	7.4579e+0 (1.69e+1) -	1.4490e+0 (5.78e-2) =	1.4547e+0 (5.29e-2) =	9.5502e+0 (2.46e+1) =	1.4413e+0 (5.37e-2) =	1.4369e+0 (4.74e-2) =	1.4365e+0 (3.34e-2) =	1.4434e+0 (5.69e-2)
SDC5	1.5544e+3 (2.76e+2) -	1.0727e+3 (1.77e+2) =	1.1229e+3 (2.23e+2) =	1.6489e+3 (2.06e+2) -	1.0977e+3 (2.30e+2) =	1.15772e+3 (1.06e+2) =	1.1656e+3 (1.56e+2) =	1.1389e+3 (2.05e+2)
SDC6	1.4818e+3 (1.33e+3) -	5.3077e+2 (4.63e+2) =	7.4113e+2 (4.06e+2) -	1.6124e+3 (1.14e+3) -	5.2914e+2 (5.18e+2) =	5.4726e+2 (7.29e+2) =	3.7932e+2 (3.76e+2) =	3.4798e+2 (3.57e+2)
SDC7	8.6017e+0 (7.33e-1) -	1.5762e+0 (2.35e+0) +	1.6438e+0 (3.92e-1) +	7.9486e+0 (1.16e+0) -	1.3752e+0 (2.92e-1) +	1.5655e+0 (5.45e-1) -	1.5918e+0 (4.39e-1) +	2.1953e+0 (2.83e-1)
SDC8	1.2147e+2 (2.06e+1) -	7.4073e+0 (7.49e+0) =	4.8415e+0 (4.53e+0) =	1.4722e+2 (6.42e+1) -	9.9778e+0 (1.36e+1) =	8.6549e+0 (1.10e+1) =	6.5532e+0 (8.42e+0) =	7.8429e+0 (8.68e+0)
SDC9	NaN (40%) -	2.1172e+2 (7.81e+1) +	2.5183e+2 (3.84e+1) +	NaN (60%) -	2.3560e+2 (9.59e+1) +	2.9962e+2 (1.28e+2) +	2.4543e+2 (1.12e+2) +	4.5855e+2 (9.38e+1)
SDC10	1.1512e+1 (3.95e-1) =	1.0395e+1 (9.82e-1) =	9.9930e+0 (5.19e-1) =	1.0237e+1 (6.72e-1) =	1.0070e+1 (1.96e-1) =	9.9195e+0 (4.59e-1) =	1.0078e+1 (3.08e+1) =	9.9617e+0 (2.66e-1)
SDC11	7.1169e+2 (9.70e+1) -	4.6232e+2 (1.22e+2) =	3.8439e+2 (1.10e+2) =	7.9998e+2 (1.50e+2) -	4.2705e+2 (8.04e+1) =	4.2251e+2 (7.40e+1) =	3.8789e+2 (1.02e+2) =	4.0670e+2 (8.43e+1)
SDC12	2.0321e+2 (6.32e+1) -	8.3649e+0 (4.70e+0) =	1.0663e+1 (4.90e+0) =	2.3262e+2 (4.18e+1) -	8.8645e+0 (5.12e+0) =	8.8029e+0 (4.23e+0) =	9.8084e+0 (6.86e+0) =	8.3027e+0 (7.46e+0)
SDC13	NaN (12%) -	1.3276e+1 (1.28e+0) =	1.2687e+1 (4.98e-1) =	NaN (12%) -	1.2317e+1 (4.18e-1) =	1.2463e+1 (3.05e-1) =	1.2504e+1 (3.59e-1) =	1.2645e+1 (7.26e-1)
SDC14	NaN (0%) -	NaN (72%) -	NaN (72%) -	NaN (0%) -	NaN (80%) =	NaN (80%) =	NaN (80%) =	NaN (80%) =
SDC15	1.1476e+1 (1.25e+0) -	1.0121e+1 (5.78e-2) =	1.0140e+1 (1.02e-1) =	1.1827e+1 (2.30e+0) =	1.0093e+1 (4.00e-2) =	1.0128e+1 (8.81e-2) =	1.0122e+1 (7.89e-2) =	1.0083e+1 (4.64e-2)
ZXH_CFI	4.0102e-2 (7.84e-4) =	4.0542e-2 (8.72e-4) =	4.0109e-2 (9.56e-4) =	4.0800e-2 (1.12e-3) =	4.1329e-2 (1.06e-3) =	4.0534e-2 (1.02e-3) =	4.0448e-2 (1.27e-3) =	4.0332e-2 (1.21e-3)
ZXH_CF2	3.9983e-1 (3.34e-1) =	2.4599e-1 (4.80e-1) =	1.8472e-1 (1.81e-1) =	4.7748e-1 (6.61e-1) =	2.4697e-1 (2.65e-1) =	5.4825e-1 (5.64e-1) -	4.3822e-1 (5.20e-1) =	1.6561e-1 (2.76e-1)
ZXH_CF3	5.9686e-2 (2.16e-3) =	6.0340e-2 (2.66e-3) =	6.2030e-2 (1.93e-3) -	6.0594e-2 (1.26e-3) -	6.0130e-2 (2.16e-3) =	6.0003e-2 (1.80e-3) =	6.1415e-2 (2.57e-3) -	5.8004e-2 (2.98e-3)
ZXH_CF4	4.9124e-2 (5.27e-2) =	3.3259e-2 (2.21e-3) =	3.3134e-2 (8.82e-4) =	3.3413e-2 (1.46e-3) =	3.3265e-2 (1.66e-3) =	3.3102e-2 (1.57e-3) =	3.4037e-2 (1.11e-3) =	3.3352e-2 (1.70e-3)
ZXH_CF5	1.7743e-1 (1.96e-1) =	1.4928e-1 (1.41e-1) =	1.1970e-1 (1.27e-1) =	2.3047e-1 (3.38e-1) =	2.5920e-1 (3.63e-1) =	3.3845e-1 (4.50e-1) =	1.8987e-1 (1.77e-1) =	7.1258e-2 (9.07e-2)
ZXH_CF6	2.4763e-2 (1.14e-3) -	2.4920e-2 (6.74e-4) -	2.4678e-2 (6.81e-4) -	2.4958e-2 (9.05e-4) -	2.4412e-2 (8.83e-4) -	2.4920e-2 (9.09e-4) -	2.4326e-2 (6.82e-4) -	2.3469e-2 (5.39e-4)
ZXH_CF7	2.2446e-2 (6.31e-4) -	3.1507e-2 (2.95e-2) -	2.2332e-2 (5.16e-4) -	2.2037e-2 (6.83e-4) -	2.2596e-2 (6.99e-4) -	2.2019e-2 (2.98e-2) =	2.2723e-2 (1.02e-3) -	2.1831e-2 (7.05e-4)
ZXH_CF8	3.8791e-2 (8.11e-4) -	3.8330e-2 (1.55e-3) -	3.8760e-2 (7.61e-4) -	3.8457e-2 (1.21e-3) -	3.8983e-2 (1.13e-3) -	3.8185e-2 (8.12e-4) -	3.8689e-2 (1.14e-3) -	3.5995e-2 (1.31e-3)
ZXH_CF9	1.4775e-2 (7.52e-4) -	1.4187e-2 (7.25e-4) =	1.4728e-2 (7.73e-4) =	1.4861e-2 (9.86e-4) -	1.4449e-2 (6.65e-4) -	1.5074e-2 (6.25e-4) -	1.5019e-2 (6.24e-4) -	1.4022e-2 (5.62e-4)
ZXH_CF10	1.2514e-2 (4.31e-4) =	1.2505e-2 (5.48e-4) =	1.2632e-2 (3.72e-4) =	1.2787e-2 (4.20e-4) -	1.2559e-2 (4.00e-4) =	1.2326e-2 (3.68e-4) =	1.2425e-2 (3.65e-4) =	1.2401e-2 (2.86e-4)
ZXH_CF11	9.2343e-3 (3.03e-4) =	9.3437e-3 (6.41e-4) =	9.2428e-3 (4.50e-4) =	8.9708e-3 (2.09e-4) =	9.1720e-3 (4.44e-4) =	9.2137e-3 (3.06e-4) =	9.0702e-3 (2.36e-4) =	9.0386e-3 (2.57e-4)
ZXH_CF12	9.1379e-2 (9.21e-2) =	2.1698e-1 (2.56e-1) =	2.8074e-1 (4.02e-1) =	1.2165e-1 (1.81e-1) =	1.1838e-1 (1.45e-1) =	1.0336e-1 (1.57e-1) =	1.8001e-1 (2.35e-1) =	1.6745e-1 (2.01e-1)
ZXH_CF13	2.8925e-2 (9.69e-2) -	1.1282e-3 (2.87e-5) =	1.1245e-3 (3.03e-5) +	1.0970e-3 (3.64e-5) +	3.5434e-2 (7.24e-2) =	1.1125e-3 (3.21e-5) +	1.1182e-3 (2.65e-5) +	1.1719e-3 (5.62e-5)
ZXH_CF14	2.1743e-3 (4.57e-5) =	2.1577e-3 (5.72e-5) =	2.1739e-3 (3.95e-5) =	2.2038e-3 (6.41e-5) =	2.1667e-3 (4.22e-5) =	2.1954e-3 (6.64e-5) =	2.2306e-3 (4.27e-5) -	2.1886e-3 (5.52e-5)
ZXH_CF15	7.7918e-2 (1.40e-1) =	2.5256e-1 (2.37e-1) =	1.9836e-1 (2.83e-1) =	1.4167e-1 (2.09e-1) =	2.6843e-1 (2.45e-1) =	1.5792e-1 (2.49e-1) =	2.0935e-1 (2.80e-1) =	6.3026e-2 (1.07e-1)
ZXH_CF16	1.1276e-3 (3.30e-5) =	1.1478e-3 (3.09e-5) =	1.1305e-3 (2.18e-5) =	1.1387e-3 (2.72e-5) =	1.1439e-3 (3.11e-5) =	1.1431e-3 (3.03e-5) =	1.1449e-3 (4.23e-5) =	1.1188e-3 (4.88e-5)

+/-=

0/15/16

2/4/25

3/6/23

1/15/15

2/4/25

3/4/24

3/6/22

TABLE S-6
MEAN HV RESULTS FOR DIFFERENT COMBINATIONS OF PARAMETER K AND PE

Problem	DVCEA_5_2	DVCEA_5_6	DVCEA_5_11	DVCEA_10_2	DVCEA_10_4	DVCEA_10_6	DVCEA_10_11	DVCEA_5_4
	$K=5$	$K=5$	$K=5$	$K=10$	$K=10$	$K=10$	$K=10$	$K=5$
	$PE=2$	$PE=6$	$PE=11$	$PE=2$	$PE=4$	$PE=6$	$PE=11$	$PE=4$
SDC1	1.1085e+00 (3.09e-02) -	1.1227e+00 (5.20e-03) =	1.1227e+00 (6.92e-03) =	1.1218e+00 (2.59e-02) -	1.1248e+00 (3.70e-03) =	1.1227e+00 (4.14e-03) =	1.1238e+00 (7.97e-03) =	1.1287e+00 (3.67e-03)
SDC2	1.2100e+00 (1.66e-05) =	1.2100e+00 (9.53e-06) =	1.2100e+00 (9.72e-06) =	1.2100e+00 (1.16e-05) =	1.2100e+00 (2.24e-05) =	1.2100e+00 (9.03e-06) =	1.2100e+00 (8.80e-06) =	1.2100e+00 (1.89e-07)
SDC3	1.0377e+00 (6.43e-03) +	1.0249e+00 (1.01e-02) =	1.0312e+00 (1.02e-02) =	1.0384e+00 (8.94e-03) +	1.0215e+00 (1.18e-02) =	1.0268e+00 (9.82e-03) =	1.0285e+00 (5.48e-03) =	1.0279e+00 (1.22e-02)
SDC4	1.2091e+00 (2.68e-03) -	1.2100e+00 (7.86e-07) =	1.2100e+00 (7.58e-07) =	1.2082e+00 (5.48e-03) -	1.2100e+00 (7.77e-07) =	1.2100e+00 (6.85e-07) =	1.2100e+00 (4.63e-07) =	1.2100e+00 (8.24e-07)
SDC5	1.2055e+00 (2.13e-03) -	1.2076e+00 (2.26e-03) =	1.2076e+00 (2.62e-03) =	1.2058e+00 (2.33e-03) -	1.2085e+00 (1.21e-03) =	1.2075e+00 (1.97e-03) =	1.2081e+00 (8.71e-04) =	1.2083e+00 (6.35e-04)
SDC6	1.1877e+00 (3.67e-02) -	1.2063e+00 (5.50e-03) -	1.2039e+00 (5.39e-03) -	1.1878e+00 (2.66e-02) -	1.2040e+00 (6.56e-03) =	1.1932e+00 (4.37e-02) =	1.1992e+00 (2.02e-02) =	1.2085e+00 (2.59e-03)
SDC7	9.6452e-01 (3.54e-02) -	1.1815e+00 (4.37e-03) +	1.1819e+00 (7.20e-03) +	9.9465e-01 (5.11e-02) -	1.1815e+00 (4.78e-03) +	1.1789e+00 (1.01e-02) =	1.1808e+00 (9.84e-03) +	1.1717e+00 (5.15e-03)
SDC8	1.2077e+00 (9.67e-04) -	1.2099e+00 (6.60e-05) =	1.2100e+00 (2.46e-05) =	1.2067e+00 (1.67e-03) -	1.2099e+00 (8.24e-05) =	1.2099e+00 (1.43e-04) =	1.2099e+00 (4.13e-05) =	1.2099e+00 (7.67e-05)
SDC9	Nan (40%) -	1.1915e+00 (8.88e-03) -	1.1882e+00 (1.13e-02) =	Nan (60%) -	1.1889e+00 (1.38e-02) =	1.1881e+00 (1.47e-02) =	1.1839e+00 (1.54e-02) =	1.1965e+00 (9.91e-03)
SDC10	8.2664e-01 (1.48e-01) -	9.1130e-01 (8.70e-02) =	9.5418e-01 (9.67e-03) =	9.0556e-01 (7.63e-02) =	9.3417e-01 (2.85e-02) =	9.4688e-01 (3.58e-02) =	9.2403e-01 (2.83e-02) =	9.4535e-01 (2.67e-02)
SDC11	1.1966e+00 (4.13e-03) -	1.2039e+00 (2.35e-03) =	1.2051e+00 (4.14e-03) =	1.1937e+00 (5.53e-03) -	1.2040e+00 (2.00e-03) =	1.2044e+00 (2.01e-03) =	1.2039e+00 (3.71e-03) =	1.2054e+00 (1.99e-03)
SDC12	1.1972e+00 (5.84e-03) -	1.2098e+00 (1.20e-04) =	1.2099e+00 (6.25e-05) =	1.1955e+00 (4.56e-03) -	1.2099e+00 (7.35e-05) =	1.2098e+00 (1.96e-04) =	1.2099e+00 (7.01e-05) =	1.2099e+00 (1.09e-04)
SDC13	Nan (12%) -	9.6200e-01 (4.38e-02) =	9.6515e-01 (3.92e-02) =	Nan (12%) -	9.6960e-01 (3.47e-02) =	9.4359e-01 (6.23e-02) =	9.6516e-01 (4.39e-02) =	9.8911e-01 (2.10e-02)
SDC14	Nan (0%) -	Nan (72%) -	Nan (72%) -	Nan (0%) -	Nan (80%) =	Nan (80%) =	Nan (80%) =	Nan (80%)
SDC15	1.3293e+00 (6.34e-04) -	1.3299e+00 (5.99e-05) =	1.3299e+00 (1.26e-04) =	1.3290e+00 (1.61e-03) -	1.3299e+00 (6.02e-05) =	1.3299e+00 (1.06e-04) =	1.3299e+00 (8.32e-05) =	1.3300e+00 (5.91e-05)
ZXH_CF1	1.3077e+00 (1.27e-04) =	1.3077e+00 (1.21e-04) =	1.3077e+00 (1.53e-04) =	1.3077e+00 (2.02e-04) =	1.3076e+00 (1.76e-04) =	1.3076e+00 (1.61e-04) =	1.3077e+00 (2.14e-04) =	1.3077e+00 (1.99e-04)
ZXH_CF2	1.2773e+00 (3.95e-02) =	1.2815e+00 (8.83e-02) =	1.3019e+00 (1.48e-02) =	1.2414e+00 (1.34e-01) =	1.2945e+00 (2.90e-02) =	1.2420e+00 (1.13e-01) =	1.2599e+00 (9.39e-02) =	1.3004e+00 (3.20e-02)
ZXH_CF3	1.2505e+00 (9.33e-04) =	1.2504e+00 (8.00e-04) =	1.2498e+00 (8.78e-04) =	1.2499e+00 (5.99e-04) =	1.2503e+00 (1.08e-03) =	1.2501e+00 (7.93e-04) =	1.2497e+00 (6.68e-04) =	1.2504e+00 (1.66e-03)
ZXH_CF4	1.2191e+00 (1.79e-02) =	1.2245e+00 (9.13e-04) =	1.2243e+00 (5.73e-04) =	1.2241e+00 (6.13e-04) =	1.2246e+00 (4.75e-04) =	1.2242e+00 (5.73e-04) =	1.2239e+00 (6.17e-04) =	1.2240e+00 (6.85e-04)
ZXH_CF5	1.1807e+00 (9.75e-02) =	1.1934e+00 (7.24e-02) =	1.2088e+00 (6.51e-02) =	1.1542e+00 (1.67e-01) =	1.1407e+00 (1.78e-01) =	1.1031e+00 (2.17e-01) =	1.1716e+00 (8.68e-02) =	1.2340e+00 (4.57e-02)
ZXH_CF6	8.0052e-01 (5.17e-03) =	8.0054e-01 (3.58e-03) =	8.0218e-01 (2.82e-03) =	8.0070e-01 (3.45e-03) =	8.0167e-01 (4.16e-03) =	8.0028e-01 (3.53e-03) =	8.0083e-01 (5.28e-03) =	8.0481e-01 (2.51e-03)
ZXH_CF7	1.2763e+00 (4.13e-03) -	1.2712e+00 (2.10e-02) -	1.2789e+00 (4.54e-03) =	1.2792e+00 (5.42e-03) =	1.2790e+00 (5.43e-03) =	1.2715e+00 (1.91e-02) =	1.2794e+00 (5.41e-03) =	1.2814e+00 (3.25e-03)
ZXH_CF8	1.0737e+00 (6.29e-03) =	1.0723e+00 (7.33e-03) -	1.0705e+00 (7.29e-03) -	1.0709e+00 (8.50e-03) -	1.0707e+00 (4.52e-03) =	1.0731e+00 (4.31e-03) =	1.0725e+00 (4.61e-03) -	1.0783e+00 (5.00e-03)
ZXH_CF9	1.1701e+00 (4.26e-04) -	1.1702e+00 (4.72e-04) -	1.1703e+00 (2.89e-04) -	1.1702e+00 (4.66e-04) -	1.1703e+00 (3.18e-04) -	1.1701e+00 (3.63e-04) -	1.1697e+00 (4.25e-04) -	1.1708e+00 (4.18e-04)
ZXH_CF10	1.1980e+00 (6.57e-04) =	1.1981e+00 (4.71e-04) =	1.1982e+00 (6.21e-04) =	1.1978e+00 (1.13e-03) =	1.1981e+00 (3.99e-04) =	1.1984e+00 (5.61e-04) =	1.1978e+00 (7.89e-04) =	1.1983e+00 (5.14e-04)
ZXH_CF11	1.0020e+00 (1.22e-03) =	1.0013e+00 (2.02e-03) =	1.0016e+00 (1.30e-03) =	1.0023e+00 (1.20e-03) =	1.0018e+00 (1.84e-03) =	1.0012e+00 (1.07e-03) =	1.0017e+00 (1.22e-03) =	1.0017e+00 (1.62e-03)
ZXH_CF12	1.3068e+00 (6.56e-03) =	1.3021e+00 (2.78e-02) =	1.2858e+00 (6.87e-02) =	1.3128e+00 (1.68e-02) =	1.3136e+00 (1.35e-02) =	1.3147e+00 (1.47e-02) =	1.3063e+00 (2.65e-02) =	1.3088e+00 (1.94e-02)
ZXH_CF13	1.0652e+00 (5.13e-05) =	1.0652e+00 (4.15e-05) =	1.0652e+00 (5.58e-05) =	1.0652e+00 (6.60e-05) =	1.0543e+00 (2.30e-02) =	1.0652e+00 (6.04e-05) =	1.0652e+00 (4.49e-05) =	1.0652e+00 (3.36e-05)
ZXH_CF14	8.5871e-01 (2.67e-04) -	8.5886e-01 (1.78e-04) =	8.5876e-01 (4.08e-04) =	8.5875e-01 (2.18e-04) -	8.5882e-01 (2.16e-04) =	8.5888e-01 (2.35e-04) =	8.5865e-01 (3.12e-04) -	8.5894e-01 (1.23e-04)
ZXH_CF15	1.1493e+00 (3.12e-02) =	1.1269e+00 (6.02e-02) =	1.1152e+00 (8.92e-02) =	1.1331e+00 (5.50e-02) =	1.1018e+00 (6.55e-02) =	1.1276e+00 (6.23e-02) =	1.1117e+00 (8.00e-02) =	1.1532e+00 (2.14e-02)
ZXH_CF16	1.1023e+00 (9.93e-05) =	1.1022e+00 (7.98e-05) =	1.1022e+00 (1.21e-04) =	1.1022e+00 (1.05e-04) =	1.1023e+00 (7.57e-05) =	1.1022e+00 (1.23e-04) =	1.1022e+00 (1.31e-04) =	1.1023e+00 (9.38e-05)

TABLE S-7
THE HV VALIDATION EXPERIMENTAL RESULTS OF DECISION VARIABLES CLASSIFICATION METHOD

Problem	DVCEA_NCP	DVCEA_NCB	DVCEA_NC_PB	DVCEA_RA	DVCEA_LM	DVCEA
SDC1	1.1152e+00 (7.92e-03) -	1.1285e+00 (2.38e-02) -	1.1200e+00 (1.77e-02) =	1.1194e+00 (2.27e-02) -	1.1317e+00 (7.32e-03) =	1.1287e+00 (3.67e-03)
SDC2	1.2100e+00 (1.34e-08) -	1.2100e+00 (7.53e-08) =	1.2100e+00 (3.36e-08) -	1.2100e+00 (1.29e-06) =	1.2100e+00 (5.89e-07) -	1.2100e+00 (1.89e-07)
SDC3	9.9856e-01 (2.07e-02) -	1.0277e+00 (1.04e-02) =	1.0139e+00 (4.41e-03) -	9.9448e-01 (2.76e-02) -	1.0277e+00 (1.07e-02) =	1.0279e+00 (1.22e-02)
SDC4	1.2100e+00 (1.28e-06) -	1.2098e+00 (3.55e-04) -	1.2100e+00 (1.62e-06) -	1.2095e+00 (9.62e-04) -	1.2091e+00 (2.15e-03) -	1.2100e+00 (8.24e-07)
SDC5	1.2066e+00 (2.02e-03) -	1.2081e+00 (8.91e-04) =	1.2090e+00 (4.10e-04) +	1.2066e+00 (1.46e-03) -	1.2080e+00 (1.08e-03) =	1.2083e+00 (6.35e-04)
SDC6	1.1970e+00 (2.31e-02) =	1.1983e+00 (2.16e-02) -	1.2068e+00 (7.61e-03) =	1.1962e+00 (2.40e-02) -	NaN (92%) -	1.2085e+00 (2.59e-03)
SDC7	1.0997e+00 (2.66e-02) -	1.1596e+00 (5.27e-03) -	1.1704e+00 (8.65e-03) =	1.1335e+00 (1.30e-02) -	1.1467e+00 (2.75e-02) -	1.1717e+00 (5.15e-03)
SDC8	1.2098e+00 (2.62e-04) -	1.2093e+00 (3.79e-04) -	1.2098e+00 (2.29e-04) -	1.2096e+00 (2.19e-04) -	1.2096e+00 (4.24e-04) -	1.2099e+00 (7.67e-05)
SDC9	1.1509e+00 (2.52e-02) -	1.1824e+00 (3.08e-02) =	1.1892e+00 (7.20e-03) -	1.1749e+00 (2.67e-02) -	NaN (92%) -	1.1965e+00 (9.91e-03)
SDC10	9.2318e-01 (4.11e-02) =	8.9718e-01 (1.09e-01) =	8.9649e-01 (6.19e-02) -	9.1084e-01 (1.15e-01) =	9.1832e-01 (5.72e-02) =	9.4535e-01 (2.67e-02)
SDC11	1.2053e+00 (2.97e-03) =	1.2008e+00 (3.53e-03) -	1.2051e+00 (2.30e-03) =	1.2037e+00 (5.03e-03) =	1.1981e+00 (6.40e-03) -	1.2054e+00 (1.99e-03)
SDC12	1.2095e+00 (5.65e-04) -	1.2076e+00 (8.65e-04) -	1.2096e+00 (2.61e-04) -	1.2083e+00 (1.13e-03) -	1.2089e+00 (1.29e-03) -	1.2099e+00 (1.09e-04)
SDC13	9.6005e-01 (2.78e-02) -	NaN (80%) -	9.6397e-01 (3.40e-02) =	9.9405e-01 (1.20e-02) =	9.7889e-01 (1.90e-02) =	9.8911e-01 (2.10e-02)
SDC14	NaN (52%) -	NaN (12%) -	NaN (32%) -	NaN (32%) -	NaN (40%) -	NaN (80%)
SDC15	1.3294e+00 (3.15e-04) -	1.3301e+00 (6.26e-05) +	1.3299e+00 (1.07e-04) -	1.3300e+00 (4.24e-05) =	1.3300e+00 (1.96e-04) =	1.3300e+00 (5.91e-05)
ZXH_CF1	1.3047e+00 (5.08e-04) -	1.3074e+00 (2.47e-04) -	1.3068e+00 (2.64e-04) -	1.3075e+00 (4.67e-04) -	1.3075e+00 (1.95e-04) -	1.3077e+00 (1.99e-04)
ZXH_CF2	1.3068e+00 (9.07e-03) =	1.2774e+00 (1.01e-01) =	1.2532e+00 (1.01e-01) -	1.2565e+00 (1.24e-01) =	NaN (92%) -	1.3004e+00 (3.20e-02)
ZXH_CF3	1.2338e+00 (1.38e-03) -	1.2498e+00 (1.50e-03) =	1.2447e+00 (1.22e-03) -	1.2473e+00 (1.85e-03) -	1.2500e+00 (1.95e-03) =	1.2504e+00 (1.66e-03)
ZXH_CF4	1.2130e+00 (1.30e-03) -	1.2193e+00 (4.27e-03) -	1.2219e+00 (5.71e-04) -	1.2193e+00 (3.17e-03) -	1.2079e+00 (2.37e-02) -	1.2240e+00 (6.85e-04)
ZXH_CF5	1.0875e+00 (1.52e-01) -	1.0608e+00 (2.60e-01) -	1.1020e+00 (2.10e-01) -	1.1977e+00 (8.56e-02) =	1.1389e+00 (2.01e-01) =	1.2340e+00 (4.57e-02)
ZXH_CF6	7.7279e-01 (4.72e-03) -	8.0649e-01 (4.32e-03) =	7.9647e-01 (3.00e-03) -	8.0188e-01 (9.50e-03) =	8.1646e-01 (2.03e-03) +	8.0481e-01 (2.51e-03)
ZXH_CF7	1.2729e+00 (3.92e-03) -	1.2463e+00 (4.84e-02) -	1.2798e+00 (3.23e-03) =	1.2741e+00 (5.18e-03) -	1.2761e+00 (2.85e-02) =	1.2814e+00 (3.25e-03)
ZXH_CF8	1.0388e+00 (5.12e-03) -	1.0745e+00 (6.34e-03) =	1.0673e+00 (8.24e-03) -	1.0666e+00 (9.09e-03) -	1.0959e+00 (4.84e-03) +	1.0783e+00 (5.00e-03)
ZXH_CF9	1.1619e+00 (8.87e-04) -	1.1692e+00 (4.49e-04) -	1.1691e+00 (4.20e-04) -	1.1686e+00 (9.15e-04) -	1.1727e+00 (4.79e-04) +	1.1708e+00 (4.18e-04)
ZXH_CF10	1.1895e+00 (1.07e-03) -	1.1956e+00 (1.07e-03) -	1.1910e+00 (2.14e-02) -	1.1877e+00 (2.36e-02) -	1.1930e+00 (2.07e-02) -	1.1983e+00 (5.14e-04)
ZXH_CF11	9.9627e-01 (1.22e-03) -	1.0012e+00 (1.39e-03) =	1.0015e+00 (1.02e-03) =	1.0008e+00 (7.59e-04) =	1.0022e+00 (1.29e-03) =	1.0017e+00 (1.62e-03)
ZXH_CF12	1.2955e+00 (4.75e-02) =	1.3096e+00 (2.68e-02) =	1.3195e+00 (5.00e-03) =	1.2895e+00 (4.14e-02) =	NaN (92%) -	1.3088e+00 (1.94e-02)
ZXH_CF13	1.0649e+00 (5.96e-05) -	1.0648e+00 (1.48e-04) -	1.0575e+00 (2.46e-02) -	1.0592e+00 (1.72e-02) -	1.0597e+00 (1.75e-02) =	1.0652e+00 (3.36e-05)
ZXH_CF14	8.5384e-01 (3.15e-04) -	8.5780e-01 (2.47e-04) -	8.5890e-01 (1.54e-04) =	8.5703e-01 (1.02e-03) -	8.5905e-01 (3.57e-04) =	8.5894e-01 (1.23e-04)
ZXH_CF15	1.1416e+00 (4.96e-02) =	1.1517e+00 (3.38e-02) =	1.1398e+00 (6.14e-02) =	1.1426e+00 (4.30e-02) =	NaN (80%) -	1.1532e+00 (2.14e-02)
ZXH_CF16	1.1019e+00 (9.01e-05) -	1.1023e+00 (8.78e-05) =	1.1023e+00 (5.47e-05) =	1.1022e+00 (1.90e-04) =	1.1022e+00 (1.68e-04) =	1.1023e+00 (9.38e-05)
+/-=	0/25/6	1/17/13	1/19/11	0/19/12	3/15/13	

TABLE S-8
THE IGD^+ VALIDATION EXPERIMENTAL RESULTS OF THE COMBINATION OF OFFSPRING GENERATION STRATEGIES

Problem	DVCEA_pbest	DVCEA_better	DVCEA_BR_PI	DVCEA_RE	DVCEA
SDC1	1.0547e+1 (2.33e-1) -	1.0137e+1 (1.02e+0) =	9.5336e+0 (3.22e-1) +	1.0122e+1 (9.47e-2) =	1.0148e+1 (1.41e-1)
SDC2	1.2402e+0 (8.94e-1) -	4.1190e-1 (5.45e-2) =	4.7485e-1 (1.29e-1) =	4.5094e-1 (1.66e-1) =	3.8472e-1 (1.43e-1)
SDC3	1.7730e+0 (2.49e-1) -	1.3371e+0 (5.55e-2) =	1.3695e+0 (4.99e-2) =	1.3453e+0 (5.43e-2) =	1.3433e+0 (6.78e-2)
SDC4	1.3585e+0 (4.21e-2) +	1.4893e+1 (1.71e+1) -	4.2021e+1 (4.53e+1) -	1.4454e+0 (4.07e-2) =	1.4434e+0 (5.69e-2)
SDC5	1.2029e+3 (2.24e+2) =	1.3613e+3 (1.23e+2) -	1.4475e+3 (2.71e+2) -	1.0362e+3 (2.68e+2) =	1.1389e+3 (2.05e+2)
SDC6	6.1411e+2 (5.43e+2) =	1.0778e+3 (1.09e+3) -	NaN (92%) -	6.6460e+2 (9.78e+2) =	3.4798e+2 (3.57e+2)
SDC7	3.6744e+0 (7.83e-1) -	2.7903e+0 (2.15e-1) -	2.9727e+0 (2.89e-1) -	2.0854e+0 (3.76e-1) =	2.1953e+0 (2.83e-1)
SDC8	6.4503e+0 (9.44e+0) =	5.0917e+1 (1.59e+1) -	5.9659e+1 (2.17e+1) -	1.5722e+1 (1.58e+1) =	7.8429e+0 (8.68e+0)
SDC9	6.0023e+2 (1.70e+2) =	NaN (92%) -	NaN (92%) -	4.5427e+2 (1.30e+2) =	4.5855e+2 (9.38e+1)
SDC10	1.0042e+1 (2.26e-1) =	9.7937e+0 (3.46e-1) =	9.9136e+0 (5.58e-1) =	1.0142e+1 (6.88e-1) =	9.9617e+0 (2.66e-1)
SDC11	3.8704e+2 (6.79e+1) =	5.6235e+2 (1.60e+2) -	5.8206e+2 (1.05e+2) -	3.8669e+2 (1.27e+2) =	4.0670e+2 (8.43e+1)
SDC12	9.0736e+0 (1.54e+1) =	8.1469e+1 (1.72e+1) -	6.2737e+1 (1.69e+1) -	1.1807e+1 (1.15e+1) =	8.3027e+0 (7.46e+0)
SDC13	1.2684e+1 (1.91e+0) =	NaN (32%) -	NaN (60%) -	1.2658e+1 (5.99e-1) =	1.2645e+1 (7.26e-1)
SDC14	NaN (72%) -	NaN (0%) -	NaN (0%) -	NaN (60%) -	NaN (80%)
SDC15	1.0011e+1 (2.97e-2) =	9.6523e+0 (3.69e-1) =	9.6690e+0 (3.42e-1) =	1.0126e+1 (7.89e-2) =	1.0083e+1 (4.64e-2)
ZXH_CF1	NaN (96.1%) -	3.7190e-2 (1.54e-3) +	4.5420e-2 (1.18e-3) -	4.0138e-2 (1.08e-3) =	4.0332e-2 (1.21e-3)
ZXH_CF2	5.6380e-1 (5.91e-1) -	2.2707e-1 (3.39e-1) =	6.8621e-1 (5.15e-1) -	2.1488e-1 (2.20e-1) =	1.6561e-1 (2.76e-1)
ZXH_CF3	1.2946e-1 (4.53e-3) -	4.7998e-2 (1.84e-3) +	6.8974e-2 (2.23e-3) -	5.9153e-2 (3.02e-3) =	5.8004e-2 (2.98e-3)
ZXH_CF4	1.4165e-1 (2.16e-2) -	3.4617e-2 (1.36e-3) =	8.8208e-2 (7.53e-3) -	3.2773e-2 (1.18e-3) =	3.3352e-2 (1.70e-3)
ZXH_CF5	3.5550e-1 (4.31e-1) -	1.9595e-1 (2.52e-1) =	5.5897e-1 (5.54e-1) -	2.2121e-1 (2.23e-1) =	7.1258e-2 (9.07e-2)
ZXH_CF6	4.2354e-2 (2.15e-3) -	2.1228e-2 (7.15e-4) +	2.5958e-2 (1.03e-3) -	2.3936e-2 (9.24e-4) =	2.3469e-2 (5.39e-4)
ZXH_CF7	6.1161e-2 (4.82e-3) -	7.4682e-2 (1.33e-1) -	6.4274e-2 (3.42e-2) -	2.1722e-2 (8.93e-4) =	2.1831e-2 (7.05e-4)
ZXH_CF8	8.8906e-2 (3.24e-3) -	3.0829e-2 (7.61e-4) +	5.7412e-2 (2.08e-3) -	3.6564e-2 (1.31e-3) =	3.5995e-2 (1.31e-3)
ZXH_CF9	5.2215e-2 (3.14e-3) -	1.3570e-2 (4.51e-4) =	2.6082e-2 (1.11e-3) -	1.4056e-2 (4.07e-4) =	1.4022e-2 (5.62e-4)
ZXH_CF10	5.2574e-2 (5.00e-3) -	2.5580e-2 (3.80e-2) -	3.8642e-2 (4.34e-3) -	1.2437e-2 (5.05e-4) =	1.2401e-2 (2.86e-4)
ZXH_CF11	2.1599e-2 (1.45e-3) -	8.9809e-3 (2.26e-4) =	1.2357e-2 (5.95e-4) -	8.9600e-3 (1.65e-4) =	9.0386e-3 (2.57e-4)
ZXH_CF12	2.3270e-1 (2.06e-1) =	2.4916e-1 (3.28e-1) =	2.4836e-1 (2.43e-1) =	9.0571e-2 (1.38e-1) =	1.6745e-1 (2.01e-1)
ZXH_CF13	2.2155e-2 (5.47e-2) -	3.6097e-2 (7.24e-2) -	1.0586e-2 (1.81e-3) -	1.8308e-2 (5.42e-2) =	1.1719e-3 (5.62e-5)
ZXH_CF14	1.4431e-2 (9.15e-4) -	2.7221e-3 (1.32e-4) -	8.4514e-3 (4.99e-4) -	2.1716e-3 (3.77e-5) =	2.1886e-3 (5.52e-5)
ZXH_CF15	1.6276e-1 (2.39e-1) -	1.9323e-1 (2.92e-1) -	3.2544e-1 (4.93e-1) -	1.1766e-1 (8.29e-2) =	6.3026e-2 (1.07e-1)
ZXH_CF16	2.4849e-3 (7.37e-5) -	1.1357e-3 (3.61e-5) =	1.3051e-3 (4.64e-5) -	1.1371e-3 (2.18e-5) =	1.1188e-3 (4.88e-5)
+/-=	1/19/11	4/15/12	1/25/5	0/1/30	

TABLE S-9
THE *HV* VALIDATION EXPERIMENTAL RESULTS OF THE COMBINATION OF OFFSPRING GENERATION STRATEGIES

Problem	DVCEA_pbest	DVCEA_better	DVCEA_BR_PI	DVCEA_RE	DVCEA
SDC1	1.1177e+00 (5.68e-03) -	1.1282e+00 (1.69e-02) -	1.1361e+00 (5.50e-03) +	1.1294e+00 (3.38e-03) =	1.1287e+00 (3.67e-03)
SDC2	1.2100e+00 (4.79e-06) -	1.2100e+00 (5.54e-08) =	1.2100e+00 (4.98e-07) -	1.2100e+00 (1.28e-06) =	1.2100e+00 (1.89e-07)
SDC3	9.6548e-01 (4.05e-02) -	1.0268e+00 (7.51e-03) =	1.0261e+00 (8.61e-03) =	1.0289e+00 (9.12e-03) =	1.0279e+00 (1.22e-02)
SDC4	1.2100e+00 (3.30e-06) =	1.2086e+00 (2.58e-03) -	1.2001e+00 (1.37e-02) -	1.2100e+00 (6.17e-07) =	1.2100e+00 (8.24e-07)
SDC5	1.2063e+00 (2.69e-03) -	1.2075e+00 (2.00e-03) =	1.2053e+00 (2.86e-03) -	1.2087e+00 (4.85e-04) =	1.2083e+00 (6.35e-04)
SDC6	1.1979e+00 (1.74e-02) =	1.1968e+00 (2.20e-02) -	NaN (92%) -	1.2014e+00 (1.98e-02) =	1.2085e+00 (2.59e-03)
SDC7	1.1364e+00 (2.12e-02) -	1.1595e+00 (7.12e-03) -	1.1304e+00 (3.02e-02) -	1.1744e+00 (4.11e-03) =	1.1717e+00 (5.15e-03)
SDC8	1.2100e+00 (4.28e-05) =	1.2096e+00 (1.19e-04) -	1.2097e+00 (1.84e-04) -	1.2099e+00 (1.01e-04) =	1.2100e+00 (7.67e-05)
SDC9	1.1755e+00 (3.03e-02) -	NaN (92%) -	NaN (92%) -	1.1967e+00 (8.37e-03) =	1.1965e+00 (9.91e-03)
SDC10	9.5341e-01 (5.20e-03) =	9.5516e-01 (4.34e-02) =	9.4866e-01 (5.53e-02) =	9.2408e-01 (6.98e-02) =	9.4535e-01 (2.67e-02)
SDC11	1.2060e+00 (1.23e-03) =	1.2009e+00 (4.17e-03) -	1.1997e+00 (2.25e-03) -	1.2061e+00 (2.65e-03) =	1.2054e+00 (1.99e-03)
SDC12	1.2098e+00 (2.21e-04) =	1.2076e+00 (1.12e-03) -	1.2093e+00 (2.48e-04) -	1.2098e+00 (1.92e-04) =	1.2099e+00 (1.09e-04)
SDC13	9.8189e-01 (6.89e-02) =	NaN (32%) -	NaN (60%) -	9.6420e-01 (7.56e-02) =	9.8911e-01 (2.10e-02)
SDC14	NaN (72%) =	NaN (0%) -	NaN (0%) -	NaN (60%) -	NaN (80%)
SDC15	1.3301e+00 (1.01e-05) +	1.3302e+00 (9.19e-05) +	1.3302e+00 (7.77e-05) +	1.3299e+00 (9.66e-05) =	1.3300e+00 (5.91e-05)
ZXH_CF1	NaN (96.1%) -	1.3082e+00 (2.24e-04) +	1.3068e+00 (2.44e-04) -	1.3078e+00 (1.35e-04) =	1.3077e+00 (1.99e-04)
ZXH_CF2	1.2374e+00 (1.28e-01) -	1.2925e+00 (4.68e-02) =	1.2265e+00 (1.03e-01) -	1.2986e+00 (1.97e-02) =	1.3004e+00 (3.20e-02)
ZXH_CF3	1.2270e+00 (1.34e-03) -	1.2540e+00 (9.63e-04) +	1.2464e+00 (9.61e-04) -	1.2506e+00 (9.90e-04) =	1.2504e+00 (1.66e-03)
ZXH_CF4	1.1850e+00 (7.11e-03) -	1.2233e+00 (6.88e-04) -	1.2038e+00 (2.72e-03) -	1.2240e+00 (9.19e-04) =	1.2240e+00 (6.85e-04)
ZXH_CF5	1.0863e+00 (2.08e-01) -	1.1691e+00 (1.26e-01) =	9.8990e-01 (2.65e-01) -	1.1557e+00 (1.12e-01) -	1.2340e+00 (4.57e-02)
ZXH_CF6	7.5441e-01 (5.89e-03) -	8.1165e-01 (2.38e-03) +	7.9956e-01 (3.69e-03) -	8.0408e-01 (4.16e-03) =	8.0481e-01 (2.51e-03)
ZXH_CF7	1.2449e+00 (6.59e-03) -	1.2450e+00 (8.76e-02) =	1.2489e+00 (2.47e-02) -	1.2783e+00 (5.42e-03) =	1.2814e+00 (3.25e-03)
ZXH_CF8	9.9219e-01 (6.63e-03) -	1.0821e+00 (6.10e-03) =	1.0425e+00 (6.14e-03) -	1.0744e+00 (7.44e-03) =	1.0783e+00 (5.00e-03)
ZXH_CF9	1.1465e+00 (1.87e-03) -	1.1711e+00 (2.33e-04) =	1.1621e+00 (8.86e-04) -	1.1706e+00 (3.45e-04) =	1.1708e+00 (4.18e-04)
ZXH_CF10	1.1709e+00 (3.47e-03) -	1.1902e+00 (2.11e-02) -	1.1779e+00 (2.70e-03) -	1.1985e+00 (7.68e-04) =	1.1983e+00 (5.14e-04)
ZXH_CF11	9.8364e-01 (1.93e-03) -	1.0025e+00 (9.46e-04) =	9.9786e-01 (1.17e-03) -	1.0025e+00 (1.04e-03) =	1.0017e+00 (1.62e-03)
ZXH_CF12	1.3017e+00 (2.62e-02) =	1.2949e+00 (5.34e-02) =	1.2992e+00 (2.86e-02) =	1.3159e+00 (1.23e-02) =	1.3088e+00 (1.94e-02)
ZXH_CF13	1.0583e+00 (1.76e-02) -	1.0541e+00 (2.30e-02) -	1.0615e+00 (6.70e-04) -	1.0598e+00 (1.73e-02) =	1.0652e+00 (3.36e-05)
ZXH_CF14	8.4448e-01 (1.03e-03) -	8.5805e-01 (2.76e-04) -	8.5143e-01 (5.42e-04) -	8.5881e-01 (3.60e-04) =	8.5894e-01 (1.23e-04)
ZXH_CF15	1.1253e+00 (6.98e-02) =	1.1127e+00 (8.37e-02) =	1.0645e+00 (2.07e-01) -	1.1432e+00 (1.55e-02) =	1.1532e+00 (2.14e-02)
ZXH_CF16	1.1006e+00 (1.94e-04) -	1.1022e+00 (2.17e-04) =	1.1021e+00 (9.15e-05) -	1.1023e+00 (6.97e-05) =	1.1023e+00 (9.38e-05)
+/-=					
1/20/10					
4/14/13					
2/26/3					
0/2/29					

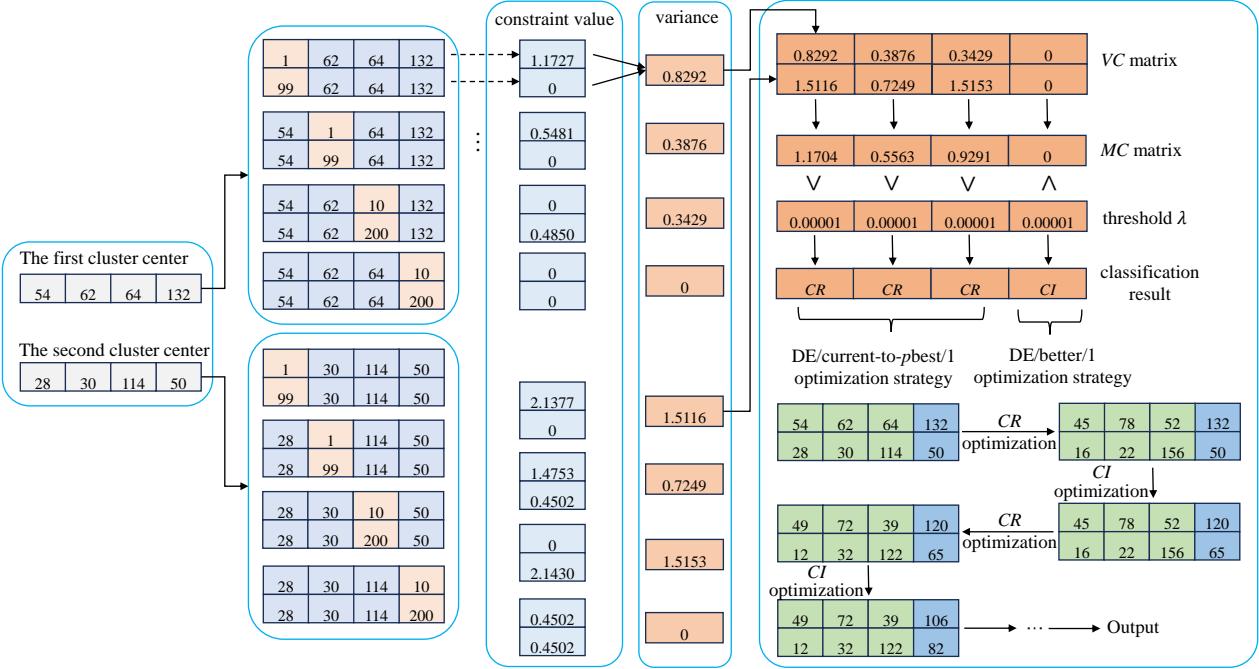


Fig. S-6. Illustration of the application of the decision variable classification method to a practical problem example.

II. THE APPLICATION OF DECISION VARIABLE CLASSIFICATION METHOD IN PRACTICAL SCENARIOS

In this section, a simple practical application case, the pressure vessel design problem, is presented to demonstrate how decision variable analysis identifies constraint-related and constraint-independent variables. The objectives and constraints of the problem are as follows:

- Objectives

$$\begin{cases} f_1 = 1.7781z_2x_3^2 + 0.6224z_1x_3x_4 + 3.1661z_1^2x_4 + 19.84z_1^2x_3 \\ f_2 = -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 \end{cases} \quad (1)$$

- Constraints

$$\begin{cases} g_1(\vec{x}) = 0.00954x_3 \leq 0.0625x_2 \\ g_2(\vec{x}) = 0.0193x_3 \leq 0.0625x_1 \end{cases} \quad (2)$$

where $1 \leq x_1, x_2 \leq 99$, $10 \leq x_3, x_4 \leq 200$. The specific classification process is shown in Fig. S-6.

1) *Selection of disturbed individuals.* K individuals are selected for decision variable perturbation. In order to avoid the situation that the new individuals generated by disturbance are close to each other and have a high degree of similarity, the initial population is first clustered, and then K cluster centers with good dispersion are selected for decision variable perturbation. In this example, assuming $K = 2$, the two cluster centers are shown in the first column of Fig. S-6.

2) *Perturbation of decision variables.* PE uniform perturbations are applied to each decision variable of K cluster centers to observe the changes in constraints. Assuming $PE = 2$, the second column shows the specific perturbation process, where each decision variable is uniformly perturbed twice, and when one variable is perturbed, the remaining variables remain unchanged. In this way, PE new individuals will be generated after each decision variable is disturbed PE times. Therefore, $K * PE * D$ new individuals will be generated after each variable of K clustering centers is disturbed, where D represents the number of decision variables.

3) *Classification of decision variables.* The degree of constraint violation for these individuals is then calculated, and the third column shows the specific constraint values. Next, for each cluster center, the variance of constraint values of the new individuals generated after each variable is perturbed is calculated, and the fourth column shows the specific variance values, then these variance values form the VC matrix. Afterward, the variance corresponding to the same dimension in all cluster centers is calculated. As a result, the MC matrix is generated, where each element in the MC corresponds to the average

variance of the constraint values for each dimension after disturbance. Finally, the classification of variables is determined by comparing the MC matrix with the threshold λ . The fourth column shows the specific comparison results. The fourth column shows the specific comparison results. The first three elements of the MC matrix are greater than the threshold λ , so the first three decision variables are constraint-related variables, and the last element is less than λ , then the last variable is a constraint-independent variable.

4) *Optimization.* After classifying decision variables, different optimization strategies are employed to optimize different decision variables. The constraint-related variables have a significant impact on the constraints, so it is hoped that the population can quickly converge and enter the feasible region by optimizing these variables. Therefore, the DE/current-to-pbest/1 strategy is employed, where each individual can achieve rapid convergence by learning from outstanding individuals in the population. In addition, when optimizing constraint-related variables, constraint-independent variables remain unchanged. However, constraint-independent variables have very little influence on constraints, so it is hoped that the search range of the population can be expanded by optimizing independent variables to increase the diversity of the population. To achieve this goal, DE/better/1 is designed where paired individuals are randomly selected, thereby increasing the learning directions of the population and promoting diversity. Similarly, constraint-related variables remain unchanged when optimizing constraint-independent variables. The lower right corner of Fig. S-6 shows the optimization process with two cluster centers as examples. When optimizing the constraint-related variables, their decision variables change from 54, 62, 64, 132, and 28, 30, 114, 50 to 45, 78, 52, 132 and 16, 22, 156, 50, respectively. At this point, DE/current-to-pbest/1 is used to generate offspring, and only constraint-related variables are changed, while constraint independent variables remain unchanged. Similarly, in the subsequent optimization of constraint-independent variables, only the constraint-independent variables are changed, from 132 and 50 to 120 and 65, respectively. Two types of variables are optimized in this alternating manner, and the final population is output when the termination condition is met.

III. THE IMPACT OF THE NUMBER OF DECISION VARIABLES ON ALGORITHM PERFORMANCE

TABLE S-10

AVERAGE IGD^+ RESULTS OBTAINED BY DVCEA AND COMPARISON ALGORITHMS ON SEVERAL SDC PROBLEMS WITH DIFFERENT NUMBERS OF DECISION VARIABLES

Problem	D	The method based on single population and single stage		The method based on two-stage		The method based on two-population		The proposed algorithm	
		NSGAIIToR	ShiP	PPS	CMOEAs_MS	IMTCMO	BiCo	DVCEA	
SDC5	100	1.7197e+3 (2.32e+2) -	1.7046e+3 (1.32e+2) -	2.0192e+3 (3.90e+2) -	1.3921e+3 (1.79e+2) -	1.7398e+3 (2.75e+2) -	1.7076e+3 (2.39e+2) -	1.1389e+3 (2.05e+2)	
	200	3.7011e+3 (3.20e+2) -	3.4144e+3 (3.50e+2) =	4.0486e+3 (5.56e+2) -	3.3626e+3 (2.61e+2) =	4.4009e+3 (5.39e+2) -	3.5030e+3 (2.63e+2) -	3.2415e+3 (1.86e+2)	
	500	9.3618e+3 (6.81e+2) =	9.2911e+3 (3.83e+2) =	1.0576e+4 (1.49e+3) -	8.5202e+3 (3.52e+2) =	1.2666e+4 (7.29e+2) -	9.3430e+3 (6.03e+2) =	8.9348e+3 (4.79e+2)	
SDC7	100	6.6283e+0 (7.73e-1) -	NaN (0%) -	1.1807e+1 (5.45e-1) -	1.0548e+1 (1.93e+0) -	4.2225e+0 (1.01e+0) -	1.2400e+1 (5.37e-1) -	2.1953e+0 (2.83e-1)	
	200	7.7614e+0 (3.35e-1) -	NaN (0%) -	1.2881e+1 (2.55e-1) -	1.1228e+1 (1.00e+0) -	7.9500e+0 (9.98e-1) -	1.2513e+1 (3.66e-1) -	2.5459e+0 (3.83e-1)	
	500	9.8872e+0 (2.01e-1) -	NaN (0%) -	1.3784e+1 (2.52e-1) -	1.2217e+1 (4.51e-1) -	1.1984e+1 (3.22e-1) -	1.2527e+1 (2.44e-1) -	4.3701e+0 (5.01e-1)	
SDC8	100	1.7979e+2 (7.97e+1) -	3.8513e+1 (1.37e+1) -	5.3891e+2 (9.83e+1) -	2.4159e+1 (1.23e+1) -	3.5584e+1 (2.45e+1) -	2.2718e+2 (5.72e+1) -	7.8429e+0 (8.68e+0)	
	200	4.4293e+2 (2.08e+2) -	1.1365e+2 (3.24e+1) -	1.2536e+3 (2.11e+2) -	7.6281e+1 (3.79e+1) -	2.1935e+2 (8.18e+1) -	6.2741e+2 (1.58e+2) -	8.5010e+0 (5.15e+0)	
	500	2.8896e+3 (8.57e+2) -	2.6444e+2 (6.68e+1) -	3.0353e+3 (5.88e+2) -	4.1732e+2 (2.97e+2) -	1.6908e+3 (3.73e+2) -	2.0716e+3 (5.17e+2) -	6.0457e+1 (3.17e+1)	
SDC11	100	7.8762e+2 (1.48e+2) -	7.0951e+2 (1.45e+2) -	8.2519e+2 (1.39e+2) -	6.3029e+2 (1.33e+2) -	3.8264e+2 (5.12e+1) =	6.0465e+2 (1.18e+2) -	4.0670e+2 (8.43e+1)	
	200	1.8709e+3 (1.86e+2) -	1.7340e+3 (1.42e+2) -	2.0550e+3 (2.41e+2) -	1.6971e+3 (7.31e+1) -	1.6385e+3 (1.60e+2) -	1.9081e+3 (2.06e+2) -	1.1774e+3 (2.11e+2)	
	500	5.4672e+3 (3.24e+2) -	4.9523e+3 (3.52e+2) =	5.6279e+3 (3.34e+2) -	4.7346e+3 (4.65e+2) =	5.0095e+3 (5.71e+2) =	5.0065e+3 (3.51e+2) =	4.7174e+3 (3.89e+2)	
SDC12	100	1.7621e+2 (4.80e+1) -	1.1114e+2 (4.23e+1) -	7.7425e+2 (8.98e+1) -	1.0131e+2 (3.99e+1) -	5.6977e+1 (2.76e+1) -	1.2497e+2 (1.92e+1) -	8.3027e+0 (7.46e+0)	
	200	5.9001e+2 (1.79e+2) -	2.7964e+2 (3.97e+1) -	1.8179e+3 (2.25e+2) -	2.3571e+2 (7.56e+1) -	3.1975e+2 (5.86e+1) -	3.4098e+2 (7.40e+1) -	1.8769e+1 (1.04e+1)	
	500	2.2017e+3 (6.16e+2) -	9.4973e+2 (2.10e+2) -	4.1264e+3 (2.94e+2) -	1.1668e+3 (4.63e+2) -	1.9364e+3 (2.46e+2) -	1.1506e+3 (1.83e+2) -	1.0178e+2 (3.03e+1)	
+/-=		0/14/1	0/12/3	0/15/0	0/12/3	0/13/2	0/13/2		

In the previous experiment, the number of decision variables is fixed, and the performance of the algorithm is observed by adjusting the ratio between constraint-related variables and constraint-independent variables. In this section, the ratio between these two types of variables is not changed, but only the total number of decision variables is increased to investigate the impact of the number of decision variables on algorithm performance. When the number of decision variables increases, the search space will also increase sharply, resulting in the algorithm can not effectively explore the search space, which puts forward a serious challenge to the performance of the algorithm. Here, SDC5, SDC7, SDC8, SDC11, and SDC12 are selected for experimentation because they use different constraint functions. In addition, several different numbers of decision variables D are set: 100, 200, and 500, and the maximum number of evaluations is set to $300,000 * D$. The experimental results of IGD^+ and HV are shown in Tables S-10 and S-11. From the IGD^+ results, DVCEA performs significantly better than the compared algorithms on 14, 12, 15, 12, 13, and 13 problems respectively, but its performance is not significantly worse than them on any problem. The average HV values also show similar results. In addition, in order to observe more clearly the impact of the number of decision variables on algorithm performance, the average IGD^+ values obtained by DVCEA and compared algorithms on several SDC problems with different numbers of decision variables are shown in Fig. S-7. As the number of decision variables increases, the IGD^+ values of all algorithms also increase, especially when D is 500, the

TABLE S-11

AVERAGE HV RESULTS OBTAINED BY DVCEA AND COMPARISON ALGORITHMS ON SEVERAL SDC PROBLEMS WITH DIFFERENT NUMBERS OF DECISION VARIABLES

Problem	D	The method based on single population and single stage			The method based on two-stage			The method based on two-population		The proposed algorithm	
		NSGAIIoR	Ship	PPS	CMOEA_MS	IMTCMO	BiCo	DVCEA			
SDC5	100	1.169e+00 (8.58e-02) -	1.1067e+00 (1.51e-01) -	1.2061e+00 (1.78e-03) -	1.1002e+00 (1.40e-01) =	1.2079e+00 (6.03e-04) =	1.2081e+00 (4.91e-04) =	1.2083e+00 (6.35e-04)			
	200	9.7740e-01 (1.34e-01) -	1.0649e+00 (1.01e-01) =	1.0902e+00 (3.32e-02) -	9.7289e-01 (9.52e-02) -	1.0724e+00 (3.12e-02) -	1.1274e+00 (1.17e-02) =	1.1294e+00 (8.63e-03)			
	500	NaN (90%) -	1.0404e+00 (1.04e-01) =	1.0265e+00 (4.54e-02) -	9.1201e-01 (7.33e-02) -	9.9292e-01 (2.64e-02) -	1.1002e+00 (1.32e-02) =	1.0933e+00 (1.02e-02)			
SDC7	100	1.0558e+00 (3.04e-02) -	NaN (0%) -	7.8345e-01 (3.77e-02) -	7.2415e-01 (2.02e-01) -	1.1287e+00 (3.03e-02) -	7.4694e-01 (3.70e-02) -	1.1717e+00 (5.15e-03)			
	200	9.8515e-01 (1.07e-01) -	NaN (0%) -	7.2926e-01 (1.94e-02) -	7.3387e-01 (1.66e-01) -	1.0037e+00 (4.54e-02) -	7.5492e-01 (2.47e-02) -	1.1722e+00 (8.10e-03)			
	500	9.3205e-01 (1.10e-02) -	NaN (0%) -	6.9841e-01 (1.46e-02) -	7.6888e-01 (1.04e-01) -	8.1746e-01 (1.86e-02) -	7.8046e-01 (1.56e-02) -	1.1342e+00 (1.41e-02)			
SDC8	100	1.2054e+00 (2.28e-03) -	1.2093e+00 (2.44e-04) -	1.1215e+00 (3.19e-02) -	1.2041e+00 (3.30e-03) -	1.2098e+00 (1.78e-04) =	1.2050e+00 (1.40e-03) -	1.2099e+00 (7.67e-05)			
	200	1.2013e+00 (6.60e-03) -	1.2087e+00 (5.41e-04) -	1.0765e+00 (2.12e-02) -	1.2011e+00 (4.89e-03) -	1.2091e+00 (7.09e-04) -	1.2011e+00 (4.56e-03) -	1.2100e+00 (5.81e-06)			
	500	1.1872e+00 (7.35e-03) -	1.2081e+00 (6.24e-04) -	1.0654e+00 (1.34e-02) -	1.1944e+00 (4.17e-03) -	1.2034e+00 (2.87e-03) -	1.1969e+00 (5.46e-03) -	1.2099e+00 (6.43e-05)			
SDC11	100	1.1965e+00 (4.81e-03) -	1.1986e+00 (5.22e-03) -	1.1945e+00 (4.84e-03) -	1.1994e+00 (6.19e-03) -	1.2065e+00 (9.41e-04) =	1.2019e+00 (3.08e-03) -	1.2054e+00 (1.99e-03)			
	200	1.1709e+00 (7.50e-03) -	1.1334e+00 (7.19e-02) -	1.1622e+00 (1.13e-02) -	1.0786e+00 (6.66e-02) -	1.1788e+00 (5.89e-03) -	1.1689e+00 (8.35e-03) -	1.1934e+00 (5.70e-03)			
	500	1.1620e+00 (5.79e-03) -	1.1562e+00 (4.89e-02) =	1.1567e+00 (6.16e-03) -	1.1569e+00 (5.78e-02) =	1.1664e+00 (1.04e-02) =	1.1693e+00 (5.60e-03) =	1.1702e+00 (6.85e-03)			
SDC12	100	1.1945e+00 (1.96e-02) -	1.2023e+00 (3.55e-03) -	1.0968e+00 (2.42e-02) -	1.1795e+00 (2.28e-02) -	1.2093e+00 (5.47e-04) -	1.2030e+00 (2.87e-03) -	1.2099e+00 (1.09e-04)			
	200	1.1851e+00 (3.02e-02) -	1.1977e+00 (6.60e-03) -	1.0647e+00 (4.20e-02) -	1.1754e+00 (2.80e-02) -	1.2066e+00 (1.32e-03) -	1.2099e+00 (2.95e-03) -	1.2099e+00 (4.99e-05)			
	500	1.1643e+00 (2.24e-02) -	1.1902e+00 (5.60e-03) -	1.0342e+00 (3.46e-02) -	1.1491e+00 (1.81e-02) -	1.1859e+00 (5.98e-03) -	1.1925e+00 (2.76e-03) -	1.2093e+00 (3.60e-04)			
$\pm/-=$		0/15/0	0/12/3	0/15/0	0/13/2	0/11/4	0/11/4				

IGD^+ values show a significant increase, indicating that the number of decision variables will greatly affect the performance of the algorithms. However, overall, DVCEA achieves the minimum IGD^+ values in all cases. The main reason for this situation is that DVCEA divides decision variables into two categories and then processes them separately, which is equivalent to dividing the high-dimensional search space into two low dimensional subspaces, reducing the dimensionality of the problem, and then searching in the low dimensional subspace. In addition, DVCEA optimizes two types of variables in a targeted manner, ensuring that the population can converge to the feasible region when optimizing constraint-related variables, and enhancing the population's exploration ability in this region when optimizing constraint-independent variables, so as to better assist the population in further locating CPF.

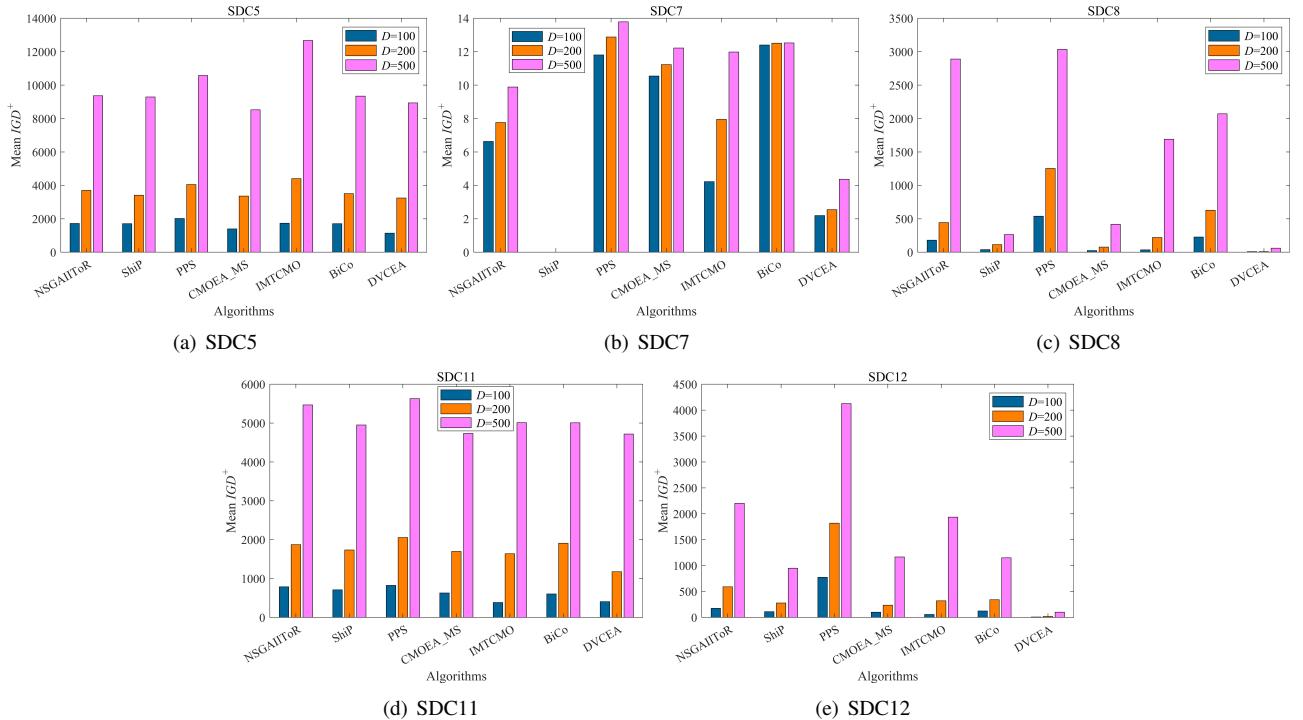


Fig. S-7. The average IGD^+ diagram of the algorithms on several SDC problems with different number of decision variables

IV. STATISTICAL ANALYSIS AND RESULTS

Fig. S-8 illustrates the population distribution of each algorithm on the SDC2 test problem. The constraint values in various regions of SDC2 are approximately equal, with the constraint values in the central region near CPF being larger than in

TABLE S-12
RESULTS OBTAINED BY THE WILCOXON TEST FOR ALGORITHM DVCEA

<i>IGD⁺</i>				
DVCEA VS	<i>R</i> ⁺	<i>R</i> ⁻	<i>P</i> -value	$\alpha \leq 0.05$
NSGAIIToR	462.0	34.0	0.000026	YES
ShiP	466.0	30.0	0.000019	YES
PPS	466.0	30.0	0.000019	YES
CMOEA_MS	443.0	22.0	0.000014	YES
IMTCMO	414.0	82.0	0.001103	YES
BiCo	452.0	44.0	0.000061	YES
<i>HV</i>				
DVCEA VS	<i>R</i> ⁺	<i>R</i> ⁻	<i>P</i> -value	$\alpha \leq 0.05$
NSGAIIToR	447.5	17.5	0.000007	YES
ShiP	460.0	5.0	0.000001	YES
PPS	463.5	1.5	0.000001	YES
CMOEA_MS	447.5	17.5	0.000007	YES
IMTCMO	437.5	27.5	0.000012	YES
BiCo	412.5	52.5	0.000133	YES

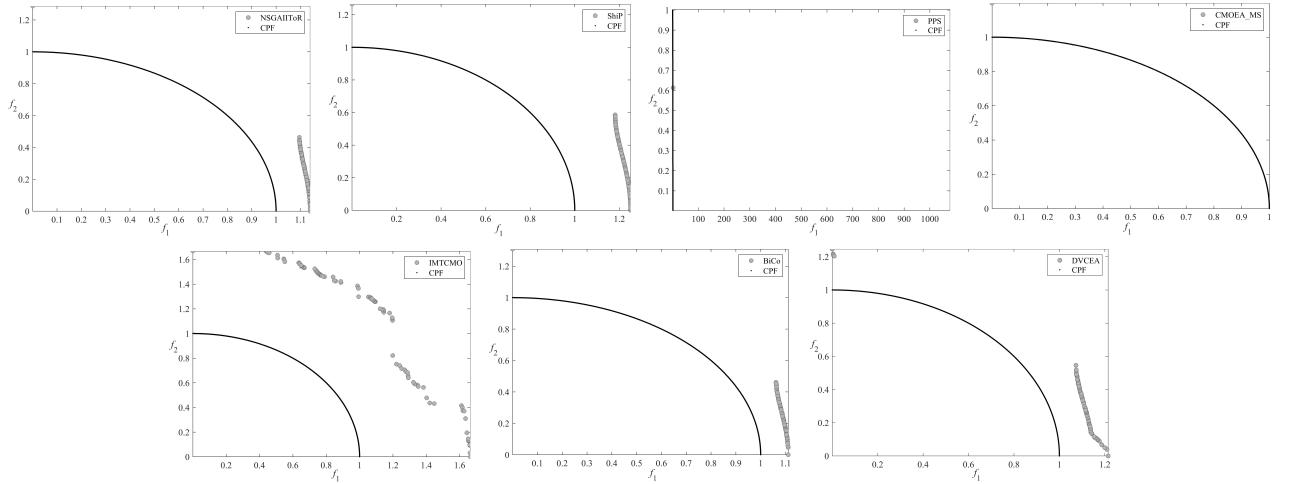


Fig. S-8. The final result distribution of each algorithm on SDC2

other regions, making it challenging to search for the CPF in the central area. From the graph, it is evident that none of the algorithms can find the complete CPF. Most algorithms converged to the upper-left and lower-right corners, with values around 1.1 or 1.2. PPS's population is trapped in a local optimum, causing some individuals to be significantly distant from the CPF. The population of CMOEA_MS converges to the same local optimal, and its diversity is poor. On the other hand, IMTCMO exhibits better population diversity, and its population distribution is superior, but its convergence is lacking, leading to a greater distance from the CPF. In addition, BiCo achieves the best results, with the population closest to CPF, while DVCEA achieves results similar to BiCo.

The final population distribution of the seven algorithms on the SDC3 test problem is shown in Fig. S-9. It is evident that multiple algorithms do not completely cover the CPF. Specifically, ShiP converges to the fourth quadrant, meaning it does not find feasible solutions. PPS, CMOEA_MS, and BiCo converge to around 3.5 and 5 on two objectives respectively, while NSGAIIToR, IMTCMO, and DVCEA converge to approximately 2.5 and 4, respectively. Furthermore, DVCEA is less than 2.5 and 4 respectively on f_1 and f_2 , indicating that DVCEA exhibits better performance than the other two algorithms.

Fig. S-10 depicts the population distribution of each algorithm on the SDC4 test problem. The CPF of SDC4 is composed of three discontinuous segments, which not only requires the algorithm to have sufficient convergence, but also sufficient diversity. From the population distribution maps, NSGAIIToR, CMOEA_MS, and BiCo do not converge to the vicinity of CPF, and are not less than 200 on both objectives. For ShiP, its population converges to the fourth quadrant, falling into the infeasible region. PPS, IMTCMO, and DVCEA all converge near CPF, but the diversity of PPS is insufficient and only a fragment of

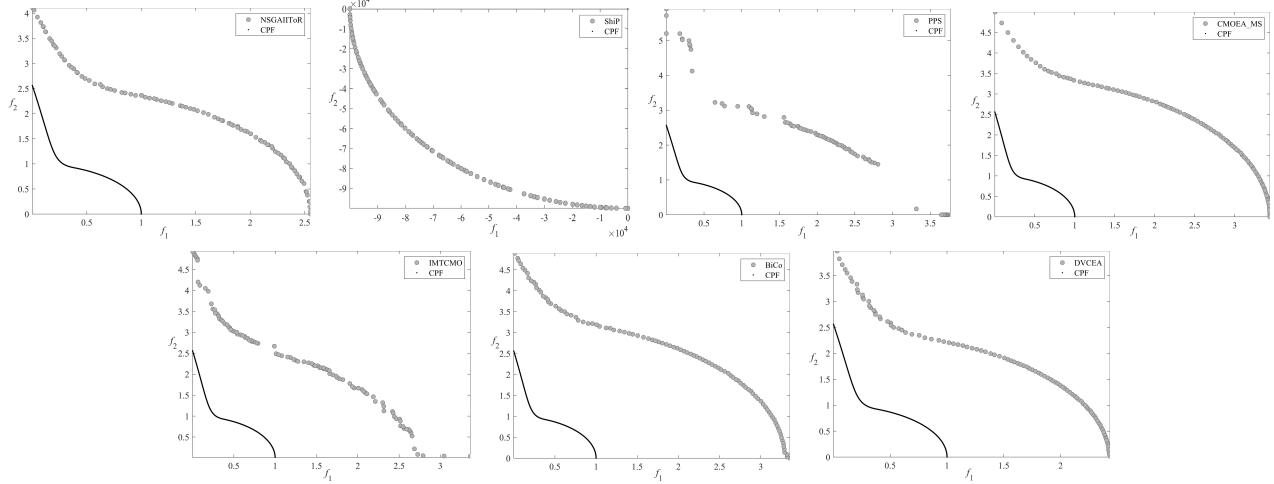


Fig. S-9. The final result distribution of each algorithm on SDC3

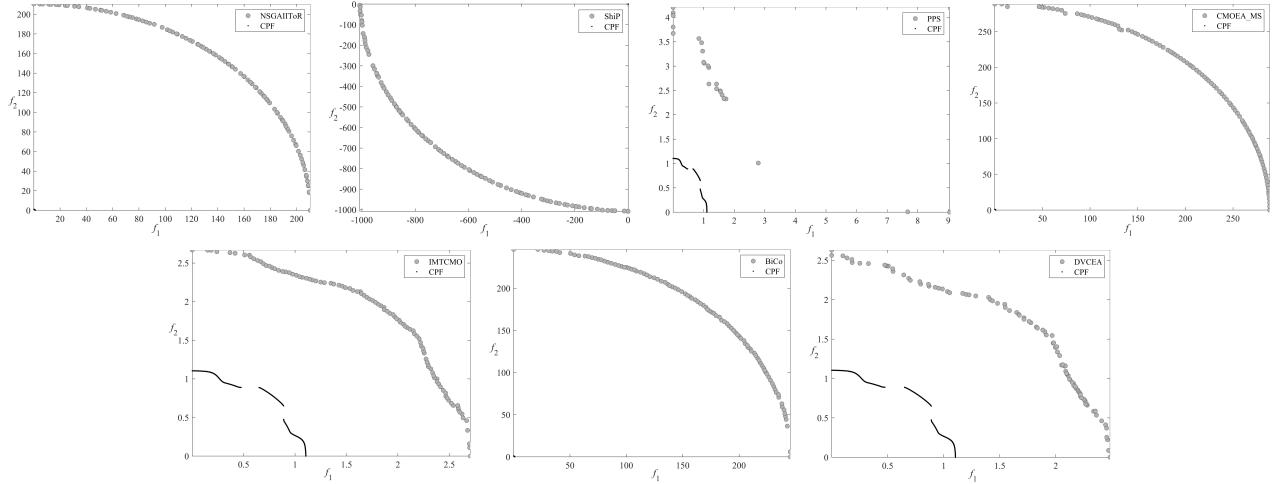


Fig. S-10. The final result distribution of each algorithm on SDC4

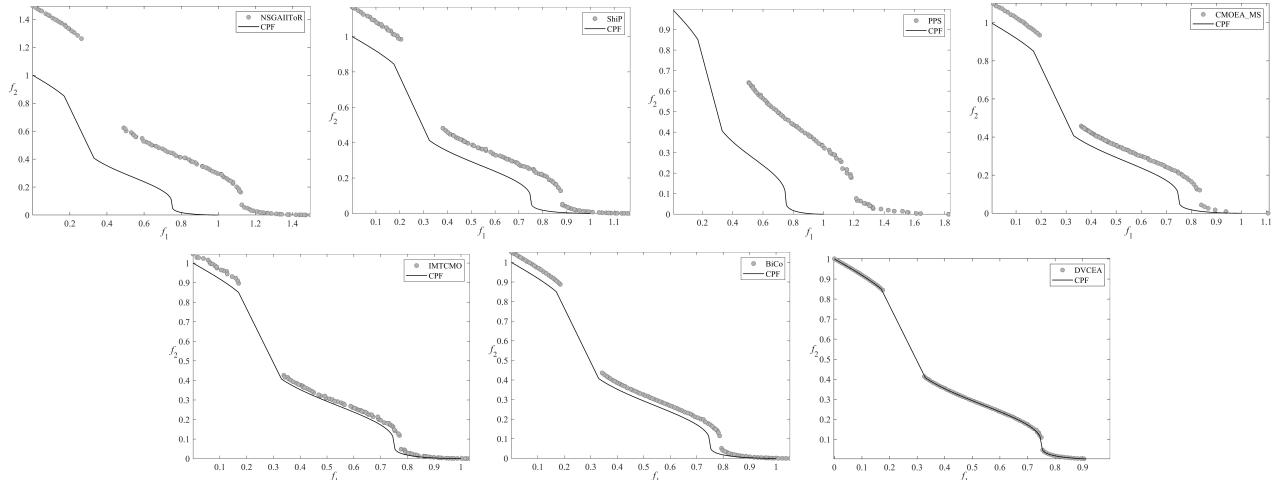


Fig. S-11. The final result distribution of each algorithm on ZXH_CF15

CPF can be searched. In contrast, the performance of IMTCMO and DVCEA is slightly better, but the two objective values of IMTCMO are greater than 2.5, while the f_1 value of DVCEA is less than 2.5, and the f_2 value converges to around 2.5, indicating that DVCEA has better performance than other algorithms.

Fig. S-11 illustrates the final population distribution for each method on the ZXH_CF15 test problem. The CPF for ZXH_CF15 consists of multiple parts. From the results of the population distributions, it can be observed that NSGAIIToR, ShiP, PPS, and CMOEA_MS do not manage to explore the CPF effectively, among which the population of NSGAIIToR is farthest from CPF. The primary reason for this is that the weight of its constraint during the search is always greater than the objective, resulting in insufficient population diversity. In addition, IMTCMO, BiCo, and DVCEA discover multiple fragments of the CPF. Specifically, IMTCMO and BiCo identify one segment, while DVCEA searches for three discontinuous segments. In general, DVCEA demonstrates the ability to explore more CPF regions, and its convergence performance is better than the comparison algorithms.