Supplementary Materials for “Multiobjective Differential Evolution for Higher-Dimensional Multimodal Multiobjective Optimization”

CONTENTS

S-I Parameter verification experiment

In HDMMODE, the parameter R is a number between 0 and 1. The purpose of this experiment is to verify the influence on the performance of the algorithm. The detailed results in terms of IGDX and IGD values obtained by HDMMODE with different *R* values on HDMMFs are listed in Table S-I-I and Table S-I-II. Overall, the performance impact of the algorithm is not significant. When the parameter *R* is set below 0.1 to 0.3, the performance of the algorithm is not greatly affected. Considering the results in the table below, this paper sets the parameter *R* to 0.2. Then, parameter validation was performed for each fitted neighborhood, verifying the number of initial neighborhood solutions denoted as *k*. In this paper, *k* was set to [3, 4, 5, 10, 20]. Experimental results indicate that the choice of *k* does not significantly impact the overall algorithm, which are list. In this paper, the value of *k* is set to 4.

TABLE S-I-I

AVERAGE **IGDX** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	*R*=0.005	*R*=0.1	*R*=0.15	*R*=0.3	*R*=0.5	*R*=0.2
HDMMF1	**1.0916e+1 (1.26e+0) =**	1.1797e+1 (2.24e+0) =	1.1283e+1 (1.86e+0) =	1.2526e+1 (2.41e+0) =	1.3819e+1 (1.65e+0) +	1.1642e+1 (2.03e+0)
HDMMF2	**7.0580e-2 (2.37e-2) =**	7.3434e-2 (2.48e-2) =	8.3074e-2 (2.50e-2) =	8.1444e-2 (4.11e-2) =	8.4108e-2 (3.05e-2) =	8.1879e-2 (2.43e-2)
HDMMF3	2.8305e+0 (2.07e-1) +	2.1947e+0 (2.00e-1) =	**2.0607e+0 (2.23e-1) -**	2.1681e+0 (3.34e-1) =	2.3009e+0 (2.90e-1) =	2.2108e+0 (2.52e-1)
HDMMF4	**3.7421e+0 (8.41e-2) -**	3.7590e+0 (8.25e-2) -	3.8688e+0 (9.44e-1) +	4.8968e+0 (2.92e+0) =	4.3852e+0 (2.17e+0) =	3.7701e+0 (1.94e-1)
HDMMF5	2.3948e+1 (2.65e+0) +	2.3864e+1 (2.06e+0) +	2.3605e+1 (2.33e+0) =	2.3139e+1 (1.86e+0) =	2.3910e+1 (2.95e+0) =	**2.2713e+1 (2.31e+0)**
HDMMF6	1.2849e+1 (2.25e+0) =	1.3272e+1 (2.93e+0) =	1.3393e+1 (3.17e+0) =	1.2790e+1 (2.53e+0) =	1.4685e+1 (3.33e+0) +	**1.2561e+1 (2.76e+0)**
HDMMF7	1.7506e+1 (1.25e+0) +	1.2080e+1 (1.91e+0) =	1.1518e+1 (1.75e+0) =	1.2114e+1 (1.76e+0) =	1.3440e+1 (2.32e+0) +	**1.1451e+1 (2.35e+0)**
HDMMF8	4.3529e+0 (6.03e-1) =	4.5925e+0 (8.59e-1) =	**4.2679e+0 (5.46e-1) =**	4.6766e+0 (8.36e-1) =	5.8028e+0 (6.47e-1) +	4.3396e+0 (6.45e-1)
HDMMF9	2.1090e+0 (4.12e-1) =	2.0244e+0 (3.56e-1) -	**2.0270e+0 (3.60e-1) -**	2.4194e+0 (3.79e-1) +	2.5368e+0 (3.58e-1) +	2.1073e+0 (3.67e-1)

HDMMF10	4.3519e+0 (2.90e+0) +	4.8385e+0 (2.92e+0) +	4.5886e+0 (2.93e+0) +	**3.1989e+0 (2.76e+0) =**	3.2250e+0 (2.48e+0) =	3.2459e+0 (3.06e+0)
HDMMF11	1.3650e+1 (1.46e+0) =	1.3604e+1 (1.29e+0) =	1.3593e+1 (1.35e+0) =	1.4119e+1 (1.19e+0) =	1.3571e+1 (1.80e+0) =	**1.3563e+1 (1.25e+0)**
HDMMF12	1.4283e+1 (1.10e+0) +	1.1024e+1 (1.05e+0) =	**1.0671e+1 (1.29e+0) =**	1.1601e+1 (1.61e+0) =	1.3077e+1 (1.65e+0) +	1.1289e+1 (1.16e+0)
HDMMF13	2.1213e+1 (7.34e-1) +	1.9952e+1 (6.30e-1) +	1.9516e+1 (7.81e-1) +	**1.8988e+1 (7.90e-1) =**	1.9177e+1 (9.88e-1) =	1.9197e+1 (6.39e-1)
HDMMF14	7.2153e+0 (6.01e-1) +	6.8636e+0 (7.96e-1) +	6.0065e+0 (7.33e-1) =	6.6799e+0 (9.25e-1) +	7.3789e+0 (1.12e+0) +	**6.0420e+0 (9.65e-1)**
HDMMF15	1.5044e+1 (1.75e+0) +	1.4955e+1 (1.72e+0) +	1.3475e+1 (1.41e+0) =	1.4317e+1 (2.22e+0) +	1.4913e+1 (2.35e+0) +	**1.3171e+1 (1.77e+0)**
+/-/=	8/1/6	5/2/8	3/2/10	3/0/12	8/0/7	

TABLE S-I-II

AVERAGE **IGD** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	*R*=0.005	*R*=0.1	*R*=0.15	*R*=0.3	*R*=0.5	*R*=0.2
HDMMF1	7.9449e+0 (1.92e+0) +	7.4078e+0 (2.05e+0) +	6.7569e+0 (2.29e+0) =	6.8249e+0 (2.55e+0) =	8.3409e+0 (2.23e+0) +	**5.9856e+0 (2.18e+0)**
HDMMF2	**3.6448e-2 (1.69e-2) -**	3.7813e-2 (2.03e-2) =	4.5310e-2 (2.25e-2) =	3.9056e-2 (2.27e-2) =	4.6073e-2 (2.09e-2) =	4.5382e-2 (2.21e-2)
HDMMF3	7.2354e-3 (8.56e-4) =	7.1409e-3 (7.51e-4) =	7.1949e-3 (7.54e-4) =	1.0140e-2 (5.78e-3) =	1.8276e-2 (1.46e-3) +	6.8555e-3 (4.90e-4)
HDMMF4	**7.1232e-3 (8.52e-4) -**	9.8842e-3 (5.05e-3) =	1.0571e-2 (4.62e-3) =	1.5916e-2 (5.44e-3) +	1.9028e-2 (4.06e-3) +	1.1146e-2 (5.14e-3)
HDMMF5	4.3458e+8 (1.19e+9) =	**1.1728e+7 (2.91e+7) =**	3.5376e+8 (1.44e+9) =	1.9474e+7 (6.11e+7) -	5.5734e+7 (1.82e+8) =	2.6574e+8 (9.57e+8)
HDMMF6	**8.0391e-1 (3.69e+0) =**	3.8017e+0 (8.73e+0) =	6.8406e+0 (1.47e+1) =	1.1195e+0 (5.07e+0) =	1.0798e+1 (4.02e+1) =	8.4542e+0 (3.69e+1)
HDMMF7	**1.7689e-2 (6.39e-3) =**	1.8417e-2 (4.62e-3) =	1.7929e-2 (9.24e-3) =	2.2453e-2 (1.16e-2) =	4.1665e-2 (5.54e-2) +	1.8012e-2 (5.81e-3)
HDMMF8	7.1003e-1 (3.90e-1) =	6.2769e-1 (3.13e-1) =	6.8249e-1 (3.01e-1) =	6.6086e-1 (3.12e-1) =	8.7529e-1 (4.64e-1) +	**5.8321e-1 (2.92e-1)**
HDMMF9	2.3789e-1 (1.42e-1) =	2.1780e-1 (1.36e-1) =	**1.9866e-1 (7.36e-2) =**	2.5289e-1 (1.34e-1) =	2.9556e-1 (1.41e-1) +	2.6443e-1 (2.20e-1)
HDMMF10	**1.0964e-2 (5.26e-3) -**	1.1450e-2 (9.49e-3) -	1.0328e-2 (3.03e-3) =	3.2577e-2 (3.13e-2) +	9.1993e-2 (1.21e-1) +	1.1782e-2 (3.78e-3)
HDMMF11	2.1150e+3 (5.79e+2) =	1.8651e+3 (5.38e+2) =	1.9989e+3 (5.34e+2) =	**1.5647e+3 (9.64e+2) =**	2.5152e+2 (9.71e+2) -	1.9609e+3 (5.50e+2)
HDMMF12	5.8416e+1 (1.30e+1) =	5.9234e+1 (8.83e+0) +	5.5872e+1 (1.13e+1) =	5.9284e+1 (9.89e+0) +	6.0497e+1 (1.62e+1) +	**5.1638e+1 (1.40e+1)**
HDMMF13	**1.7746e+4 (4.75e+3) =**	2.0129e+4 (6.17e+3) =	2.4823e+4 (9.18e+3) =	2.9610e+4 (1.69e+4) =	3.5570e+4 (2.59e+4) +	2.2234e+4 (1.04e+4)
HDMMF14	5.5058e-3 (4.22e-4) =	5.5156e-3 (4.59e-4) =	5.4242e-3 (4.35e-4) =	8.0797e-3 (4.96e-3) =	1.3866e-2 (7.82e-3) +	**5.4190e-3 (3.80e-4)**
HDMMF15	8.0110e+0 (1.30e-1) =	7.9650e+0 (2.16e-1) =	7.9941e+0 (2.02e-1) =	8.1141e+0 (1.98e-1) +	8.2073e+0 (1.87e-1) +	**7.9582e+0 (2.12e-1)**
+/-/=	1/3/11	2/1/12	0/0/15	4/1/10	11/1/3	

TABLE S-I-III

AVERAGE **IGDX** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	*k*=3	*k*=5	*k*=10	*k*=20	*k*=4
HDMMF1	1.1680e+1 (2.06e+0) =	1.2238e+1 (2.07e+0) =	1.1637e+1 (2.29e+0) =	**1.1467e+1 (1.92e+0) =**	1.1642e+1 (2.03e+0)
HDMMF2	7.8971e-2 (2.57e-2) =	**7.7786e-2 (2.92e-2) =**	8.4826e-2 (2.57e-2) =	7.9134e-2 (2.30e-2) =	8.1879e-2 (2.43e-2)
HDMMF3	2.1671e+0 (2.04e-1) =	**2.1471e+0 (2.85e-1) =**	2.1879e+0 (2.22e-1) =	2.1805e+0 (2.80e-1) =	2.2108e+0 (2.52e-1)
HDMMF4	3.7428e+0 (2.25e-1) =	3.9311e+0 (9.47e-1) =	**3.7005e+0 (2.12e-1) -**	3.8810e+0 (9.33e-1) +	3.7701e+0 (1.94e-1)
HDMMF5	2.2727e+1 (2.38e+0) =	2.2759e+1 (2.62e+0) =	2.2772e+1 (2.66e+0) =	2.3436e+1 (2.61e+0) =	**2.2713e+1 (2.31e+0)**
HDMMF6	1.2935e+1 (2.67e+0) =	1.2353e+1 (2.88e+0) =	**1.2032e+1 (2.57e+0) =**	1.3012e+1 (3.13e+0) =	1.2561e+1 (2.76e+0)
HDMMF7	1.1924e+1 (2.54e+0) =	1.1281e+1 (1.51e+0) =	**1.1085e+1 (1.63e+0) =**	1.1531e+1 (1.42e+0) =	1.1451e+1 (2.35e+0)
HDMMF8	4.3602e+0 (6.78e-1) =	4.4003e+0 (6.50e-1) =	4.5412e+0 (7.02e-1) =	4.3727e+0 (5.40e-1) =	**4.3396e+0 (6.45e-1)**
HDMMF9	**1.9321e+0 (3.17e-1) =**	2.0954e+0 (3.85e-1) =	2.0368e+0 (2.98e-1) =	2.0664e+0 (3.48e-1) =	2.1073e+0 (3.67e-1)
HDMMF10	**2.8868e+0 (2.94e+0) =**	3.1401e+0 (2.79e+0) =	3.2579e+0 (3.00e+0) =	3.5443e+0 (2.97e+0) =	3.2459e+0 (3.06e+0)
HDMMF11	**1.3134e+1 (1.41e+0) =**	1.3233e+1 (1.50e+0) =	1.3305e+1 (1.26e+0) =	1.3732e+1 (1.08e+0) =	1.3563e+1 (1.25e+0)
HDMMF12	1.1277e+1 (1.21e+0) =	1.1223e+1 (1.51e+0) =	1.0938e+1 (1.03e+0) =	**1.0882e+1 (1.24e+0) =**	1.1289e+1 (1.16e+0)
HDMMF13	1.9257e+1 (7.17e-1) =	**1.9037e+1 (9.17e-1) =**	1.9314e+1 (6.48e-1) =	1.9105e+1 (7.28e-1) =	1.9197e+1 (6.39e-1)
HDMMF14	5.7124e+0 (6.99e-1) =	**5.4943e+0 (6.17e-1) -**	5.8503e+0 (7.15e-1) =	6.0732e+0 (8.43e-1) =	6.0420e+0 (9.65e-1)
HDMMF15	**1.3002e+1 (2.06e+0) =**	1.3388e+1 (1.79e+0) =	1.3473e+1 (1.87e+0) =	1.3200e+1 (1.59e+0) =	1.3171e+1 (1.77e+0)
+/-/=	0/0/15	0/1/14	0/1/14	1/0/14	

TABLE S-I-IV

AVERAGE **IGD** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	*k*=3	*k*=5	*k*=10	*k*=20	*k*=4
HDMMF1	5.4569e+0 (2.14e+0) =	5.6542e+0 (2.21e+0) =	**5.1479e+0 (1.83e+0) =**	5.6677e+0 (1.83e+0) =	5.9856e+0 (2.18e+0)
HDMMF2	4.2604e-2 (2.26e-2) =	**4.2132e-2 (2.28e-2) =**	4.5429e-2 (2.13e-2) =	4.3557e-2 (2.22e-2) =	4.5382e-2 (2.21e-2)
HDMMF3	6.8158e-3 (4.85e-4) =	6.8403e-3 (4.78e-4) =	7.1494e-3 (5.71e-4) +	7.1482e-3 (6.87e-4) =	**6.8555e-3 (4.90e-4)**
HDMMF4	1.1231e-2 (4.89e-3) =	1.1366e-2 (4.69e-3) =	1.1573e-2 (5.23e-3) =	1.2164e-2 (4.89e-3) =	**1.1146e-2 (5.14e-3)**
HDMMF5	5.5639e+8 (1.86e+9) =	**1.6216e+8 (4.65e+8) =**	3.2923e+8 (1.22e+9) =	1.7728e+8 (5.16e+8) =	2.6574e+8 (9.57e+8)
HDMMF6	2.7086e+0 (1.11e+1) =	3.7852e+0 (9.77e+0) =	**9.5071e-1 (3.73e+0) =**	2.1506e+0 (8.49e+0) =	8.4542e+0 (3.69e+1)
HDMMF7	1.8302e-2 (5.57e-3) =	1.9418e-2 (9.56e-3) =	1.8360e-2 (7.37e-3) =	1.8077e-2 (9.83e-3) =	**1.8012e-2 (5.81e-3)**

HDMMF8	6.3672e-1 (3.02e-1) =	7.6737e-1 (2.68e-1) +	6.9750e-1 (3.39e-1) =	**5.0877e-1 (2.65e-1) =**	5.8321e-1 (2.92e-1)
HDMMF9	**1.7647e-1 (4.19e-2) =**	2.1308e-1 (1.36e-1) =	2.2501e-1 (7.81e-2) =	2.1278e-1 (5.52e-2) =	2.6443e-1 (2.20e-1)
HDMMF10	**1.0220e-2 (1.99e-3) =**	1.2164e-2 (7.59e-3) =	1.1303e-2 (3.97e-3) =	1.3085e-2 (6.25e-3) =	1.1782e-2 (3.78e-3)
HDMMF11	1.8951e+3 (7.15e+2) =	1.9291e+3 (5.68e+2) =	1.9397e+3 (5.68e+2) =	**1.8094e+3 (6.00e+2) =**	1.9609e+3 (5.50e+2)
HDMMF12	5.5145e+1 (1.41e+1) =	5.5662e+1 (1.36e+1) =	5.4934e+1 (1.25e+1) =	5.6018e+1 (1.32e+1) =	**5.1638e+1 (1.40e+1)**
HDMMF13	2.5514e+4 (1.21e+4) =	2.2827e+4 (8.85e+3) =	**2.1692e+4 (6.27e+3) =**	2.3062e+4 (8.40e+3) =	2.2234e+4 (1.04e+4)
HDMMF14	5.5415e-3 (4.00e-4) =	5.5485e-3 (2.55e-4) +	5.5468e-3 (4.86e-4) =	5.5638e-3 (3.32e-4) =	**5.4190e-3 (3.80e-4)**
HDMMF15	7.9826e+0 (1.62e-1) =	**7.9136e+0 (1.28e-1) =**	7.9706e+0 (1.39e-1) =	8.0070e+0 (1.77e-1) =	7.9582e+0 (2.12e-1)
+/-/=	0/0/15	2/0/13	1/0/14	0/0/15	

S-II Comparison with other variants

Table S-II-I and S-II-II report the detailed results of IGD and IGDX values obtained by other two variants with different large-scale handing technique. Here, HDMMODE is experimentally integrated with decision variable grouping techniques [1] and the competitive particle swarm optimization CSO operator [2], and the results show that these techniques cannot effectively solve the problem. The design purpose of the CSO operator is to further improve the convergence of the population by using a crossover and mutation approach that involves crossing and mutating solutions with half of the fitness values of the population being worse than the other half. However, this approach limits the generation of diverse and better solutions, affecting the algorithm's performance in the decision space and making it challenging to effectively solve higher-dimensional MMOPs. Experimental results also indicate that the variant HDMMODE_CSO, embedded with the CSO operator, struggles to effectively solve the problem.

[1] Ma X, Liu F, Qi Y, et al. A multiobjective evolutionary algorithm based on decision variable analyses for multiobjective optimization

problems with large-scale variables[J]. IEEE Transactions on Evolutionary Computation, 2015, 20(2): 275-298.
[2] Tian Y, Zheng X, Zhang X, et al. Efficient large-scale multiobjective optimization based on a competitive swarm optimizer[J]. IEEE Transactions on Cybernetics, 2019, 50(8): 3696-3708.

TABLE S-II-I

AVERAGE **IGDX** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	HDMMODE_CSO	HDMMODE_DVA	HDMMODE
HDMMF1	1.4284e+1 (2.80e+0) +	**8.6133e+0 (5.83e-1) -**	1.1642e+1 (2.03e+0)
HDMMF2	4.5777e+0 (6.32e-2) +	4.6075e+0 (1.46e-1) +	**8.1879e-2 (2.43e-2)**
HDMMF3	5.1904e+0 (1.15e+0) +	3.0037e+0 (5.39e-1) +	**2.2108e+0 (2.52e-1)**
HDMMF4	5.9342e+0 (3.85e+0) =	**2.6274e+0 (3.76e-2) -**	3.7701e+0 (1.94e-1)
HDMMF5	3.2647e+1 (7.19e+0) +	2.5298e+1 (1.81e+0) +	**2.2713e+1 (2.31e+0)**
HDMMF6	1.6184e+1 (5.05e+0) +	2.5000e+1 (4.04e+0) +	**1.2561e+1 (2.76e+0)**
HDMMF7	2.0714e+1 (4.73e+0) +	2.7301e+1 (3.45e+0) +	**1.1451e+1 (2.35e+0)**
HDMMF8	9.1326e+0 (1.00e+0) +	1.2845e+1 (1.11e+0) +	**4.3396e+0 (6.45e-1)**
HDMMF9	2.6263e+0 (2.70e-1) +	2.9488e+0 (9.01e-2) +	**2.1073e+0 (3.67e-1)**
HDMMF10	7.7662e+0 (4.41e-1) +	8.2182e+0 (4.21e-1) +	**3.2459e+0 (3.06e+0)**
HDMMF11	**1.0474e+1 (8.02e-1) -**	1.4813e+1 (1.76e-1) +	1.3563e+1 (1.25e+0)
HDMMF12	1.3321e+1 (1.86e+0) +	1.2138e+1 (1.43e-1) +	**1.1289e+1 (1.16e+0)**
HDMMF13	2.1085e+1 (1.12e+0) +	3.0549e+1 (1.81e+0) +	**1.9197e+1 (6.39e-1)**
HDMMF14	1.7970e+1 (2.15e+0) +	8.1840e+0 (3.98e-1) +	**6.0420e+0 (9.65e-1)**
HDMMF15	1.3098e+1 (2.72e+0) =	**1.0470e+1 (5.78e-2) -**	1.3171e+1 (1.77e+0)
+/-/=	12/1/2	12/3/0	

TABLE S-II-II

AVERAGE **IGDX** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	HDMMODE_CSO	HDMMODE_DVA	HDMMODE
HDMMF1	8.5291e+0 (2.23e+0) +	7.5699e+0 (2.01e+0) +	**5.9856e+0 (2.18e+0)**
HDMMF2	5.9576e-2 (3.58e-2) =	4.9794e+1 (2.69e+1) +	**4.5382e-2 (2.21e-2)**
HDMMF3	2.8759e-2 (1.33e-1) +	1.4274e+1 (4.83e+0) +	**6.8555e-3 (4.90e-4)**
HDMMF4	2.7073e+0 (2.92e+0) +	9.1297e+1 (1.89e+1) +	**1.1146e-2 (5.14e-3)**
HDMMF5	5.9185e+9 (1.49e+10) +	3.1354e+26 ± 1.58e+26 +	**2.6574e+8 (9.57e+8)**
HDMMF6	8.6376e+3 (4.07e+4) =	1.6324e+11 (7.89e+11) +	**8.4542e+0 (3.69e+1)**
HDMMF7	3.1095e+0 (8.03e+0) +	1.2060e+17 (3.86e+17) +	**1.8012e-2 (5.81e-3)**
HDMMF8	**6.1932e-2 (1.08e-1) -**	1.9035e+11 (4.66e+11) +	5.8321e-1 (2.92e-1)
HDMMF9	3.4049e-1 (2.56e-1) +	8.2713e+0 (1.09e+1) +	**2.6443e-1 (2.20e-1)**
HDMMF10	3.3196e+2 (1.25e+2) +	4.5228e+3 (3.44e+3) +	**1.1782e-2 (3.78e-3)**
HDMMF11	**1.5242e+3 (4.65e+2) -**	2.3095e+4 (6.02e+3) +	1.9609e+3 (5.50e+2)
HDMMF12	8.7011e+1 (2.09e+1) +	7.8309e+3 (4.15e+3) +	**5.1638e+1 (1.40e+1)**
HDMMF13	**3.7979e+3 (9.92e+2) -**	2.6402e+6 (5.80e+5) +	2.2234e+4 (1.04e+4)
HDMMF14	1.3961e-1 (1.48e-1) +	7.7770e+2 (2.88e+2) +	**5.4190e-3 (3.80e-4)**
HDMMF15	**7.5988e+0 (9.62e-1) =**	1.4561e+3 (3.36e+3) +	**7.9582e+0 (2.12e-1)**
+/-/=	9/3/3	15/0/0	

Table S-II-III report the detailed results of IGD values obtained by HDMMODE and its six variants.

TABLE S-II-III

AVERAGE **IGD** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	Variant1	Variant2	Variant3	Variant4	Variant5	Variant6	HDMMODE
HDMMF1	1.19e+1 (2.54e+0) +	1.24e+1 (3.92e+0) +	8.03e+0 (2.70e+0) +	7.81e+0 (3.80e+0) +	7.60e+0 (2.26e+0) -	**3.53e+0 (1.27e+0) -**	7.81e+0 (1.16e+0)
HDMMF2	2.11e-1 (1.25e-1) +	7.06e-2 (2.20e-2) +	**4.11e-2 (2.17e-2) -**	2.02e-2 (1.81e-3) -	3.07e-2 (4.04e-3) -	2.84e-2 (1.84e-2) -	6.27e-2 (4.28e-2)
HDMMF3	**4.11e-3 (3.42e-4) -**	1.41e-2 (4.63e-3) +	5.03e-3 (4.97e-4) -	6.82e-3 (1.39e-3) -	5.68e-3 (4.97e-4) -	9.65e-3 (4.35e-3) +	7.78e-3 (8.52e-4)
HDMMF4	1.32e+0 (1.45e+0) +	6.49e-2 (6.79e-2) +	4.60e+0 (5.39e+0) +	**5.70e-3 (2.66e-4) -**	5.01e-2 (4.06e-3) +	4.91e+0 (6.25e+0) +	3.25e-2 (6.20e-2)
HDMMF5	2.59e+9 (4.67e+9) +	3.30e+8 (3.39e+8) =	1.25e+9 (2.77e+9) +	6.63e+8 (2.02e+9) +	6.73e+7 (1.32e+8) =	9.36e+9 (1.04e+9) +	**4.51e+7 (1.90e+8)**
HDMMF6	4.75e-1 (8.24e-1) -	4.40e-2 (5.16e-2) -	4.45e-1 (1.00e+0) -	1.26e+1 (2.70e+1) =	1.43e-1 (3.62e-1) -	**4.22e-2 (3.87e-2) -**	3.16e+1 (9.26e+1)
HDMMF7	1.37e+1 (4.53e+1) +	6.84e+0 (2.27e+1) +	1.62e+1 (8.64e+1) +	2.83e-2 (1.39e-2) =	**1.88e-2 (1.23e-2) -**	4.51e+0 (2.50e+1) +	1.22e-1 (1.60e-1)
HDMMF8	**5.58e-2 (7.04e-2) -**	3.69e-1 (3.39e-1) -	1.05e+0 (3.12e-1) =	1.08e+0 (3.35e-1) +	4.60e-1 (4.22e-1) -	4.21e-1 (2.29e-1) -	1.03e+0 (1.92e-1)
HDMMF9	8.32e-1 (5.25e-1) +	7.92e-1 (4.69e-1) +	4.68e-1 (2.23e-1) +	**2.59e-1 (1.18e-1) =**	4.20e-1 (2.99e-1) +	4.73e-1 (2.25e-1) +	2.64e-1 (2.20e-1)
HDMMF10	2.68e+2 (1.75e+2) +	1.91e+1 (3.58e+1) +	3.63e+2 (3.25e+2) +	**8.49e-3 (1.59e-3) -**	3.02e-2 (3.46e-2) +	1.96e+2 (1.26e+2) +	**1.17e-2 (3.78e-3)**
HDMMF11	1.13e+3 (9.10e+2) -	**1.08e+2 (2.20e+2) -**	5.05e+2 (5.29e+2) -	1.63e+3 (4.60e+2) -	2.27e+2 (3.42e+2) -	3.72e+3 (1.46e+2) +	1.96e+3 (5.50e+2)
HDMMF12	9.91e+1 (2.84e+1) +	8.18e+1 (1.23e+1) +	8.45e+1 (3.04e+1) +	5.90e+1 (1.17e+1) +	6.27e+1 (1.17e+1) +	8.43e+1 (2.24e+1) +	**5.16e+1 (1.40e+1)**
HDMMF13	3.18e+4 (1.32e+4) +	9.66e+4 (6.85e+4) +	6.77e+4 (3.32e+4) +	**1.25e+4 (6.41e+3) -**	7.01e+4 (2.76e+4) +	6.23e+4 (2.75e+4) +	2.22e+4 (1.04e+4)
HDMMF14	1.93e-1 (1.60e-1) +	1.01e-1 (1.10e-1) +	1.10e-1 (1.29e-1) +	7.91e-3 (4.34e-4) +	8.48e-3 (1.49e-3) +	1.25e-1 (1.71e-1) +	**5.41e-3 (3.80e-4)**
HDMMF15	8.93e+0 (1.31e+0) +	1.15e+1 (8.27e-1) +	**7.30e+0 (6.57e-1) -**	9.22e+0 (5.49e-1) +	9.79e+0 (6.84e-1) +	1.11e+1 (7.36e-1) +	7.95e+0 (2.12e-1)
+/-/=	11/0/4	11/1/3	9/1/5	6/3/6	7/1/7	11/0/4	

S-III Comparison with other MMOEAs

The compared running time of different MMOEAs and the hypervolume (HV) indicators obtained by different MMOEAs on HDMMF benchmark test sets are calculated in this section. The HV indicators are shown in Table S-III-I and the compared running time results are shown in Fig. S-III-I and Table S-III-I.

TABLE S-III-I

AVERAGE **HV** VALUES OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	DNNSGAII	MMOEA/DC	MO_Ring_PSO_SCD	TriMOEATAR	MMEAWI	HREA	HDMODE
HDMMF1	1.1885e+0 (7.58e-7) +	1.2099e+0 (6.74e-6) +	1.2099e+0 (4.61e-6) +	1.1941e+0 (2.94e-7) +	1.1879e+0 (8.89e-4) +	1.1885e+0 (1.90e-8) +	**1.2099e+0 (2.25e-5)**
HDMMF2	1.2100e+0 (3.09e-7) +	1.2100e+0 (1.74e-6) +	1.2088e+0 (1.87e-4) +	1.2100e+0 (3.97e-7) +	1.2100e+0 (2.94e-7) +	1.2100e+0 (2.31e-6) +	**1.2100e+0 (1.15e-7)**
HDMMF3	1.2100e+0 (3.32e-8) -	1.2100e+0 (2.72e-6) +	1.2100e+0 (1.23e-6) +	1.2100e+0 (1.99e-8) -	**1.2100e+0 (4.01e-8) -**	1.2100e+0 (7.04e-7) +	1.2100e+0 (8.92e-8)
HDMMF4	1.2100e+0 (4.30e-8) +	1.2100e+0 (1.32e-11) +	1.2100e+0 (2.50e-6) +	1.2100e+0 (7.05e-9) +	1.2100e+0 (8.62e-9) +	1.2100e+0 (4.47e-8) +	**1.2100e+0 (2.75e-13)**
HDMMF5	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.51e-16) =	**1.2100e+0 (4.51e-16)**
HDMMF6	1.2100e+0 (4.46e-16) =	**1.2100e+0 (4.88e-16) =**	1.2100e+0 (4.25e-16) =	1.2100e+0 (4.46e-16) =	1.2100e+0 (4.40e-16) =	1.2100e+0 (3.57e-16) +	1.2100e+0 (4.42e-16)
HDMMF7	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.71e-16) =	1.2100e+0 (4.51e-16) =	1.2100e+0 (4.23e-16) =	1.2100e+0 (4.60e-16) =	1.2100e+0 (4.46e-16) =	**1.2100e+0 (1.44e-10)**
HDMMF8	1.2100e+0 (4.88e-16) =	1.2100e+0 (4.51e-16) =	**1.2100e+0 (4.90e-16) =**	1.2100e+0 (4.17e-16) =	1.2100e+0 (2.44e-12) =	1.2100e+0 (4.57e-16) =	1.2100e+0 (4.55e-16)
HDMMF9	2.3757e-1 (1.13e-16) +	7.2898e-1 (4.88e-2) +	2.6456e-1 (2.44e-7) +	2.1158e-1 (5.64e-17) +	2.3757e-1 (1.13e-16) +	2.3757e-1 (1.13e-16) +	**7.8256e-1 (1.59e-2)**
HDMMF10	1.2098e+0 (1.95e-5) +	**1.2100e+0 (5.53e-9) -**	1.2098e+0 (1.01e-4) +	1.2098e+0 (8.42e-10) +	1.2099e+0 (3.98e-7) +	1.2099e+0 (5.59e-7) +	1.2100e+0 (1.89e-8)
HDMMF11	1.0737e+0 (9.09e-3) +	**1.2099e+0 (3.43e-5) -**	1.2093e+0 (2.80e-4) +	1.1939e+0 (1.40e-2) +	1.1959e+0 (1.85e-4) +	1.1955e+0 (1.79e-4) +	1.2095e+0 (2.50e-4)
HDMMF12	1.0391e+0 (2.70e-1) +	**1.2100e+0 (8.58e-6) -**	1.2097e+0 (5.76e-5) +	1.1915e+0 (1.91e-3) +	1.1830e+0 (1.61e-4) +	1.1808e+0 (5.23e-4) +	1.2099e+0 (3.35e-4)
HDMMF13	1.3089e+0 (4.08e-2) +	1.3309e+0 (1.55e-5) +	1.3310e+0 (6.72e-6) +	1.3309e+0 (1.04e-4) +	1.3310e+0 (1.92e-7) =	1.3310e+0 (2.34e-7) =	**1.3310e+0 (5.92e-5)**
HDMMF14	6.4486e-1 (5.19e-2) +	1.2100e+0 (2.12e-6) +	1.2082e+0 (4.92e-4) +	7.4959e-1 (1.47e-1) +	8.5261e-1 (8.54e-4) +	8.5452e-1 (1.75e-4) +	**1.2100e+0 (1.10e-8)**
HDMMF15	1.1452e+0 (4.76e-2) +	**1.2100e+0 (5.14e-5) =**	1.2100e+0 (3.93e-5) =	1.0878e+0 (1.48e-2) +	1.2043e+0 (3.28e-6) +	1.2002e+0 (6.08e-3) +	1.2099e+0 (1.40e-4)
+/=/-	10/4/1	7/5/3	10/5/0	10/4/1	9/5/1	11/4/0	

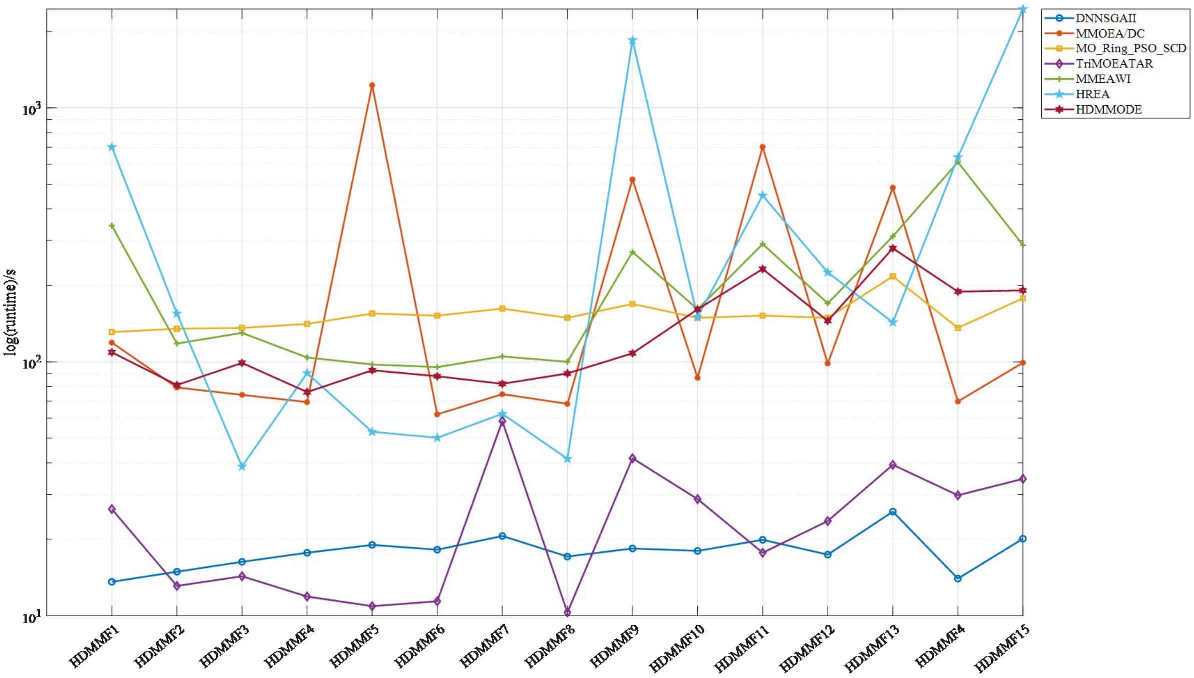

Fig. S-I. The average CPU runtime of the seven algorithms on each HDMMF test problem

TABLE S-III-II

AVERAGE **RUNNING TIME** OVER 31 RUNS ON BENCHMARK INSTANCES, WHERE THE BEST MEAN VALUE IS SHOWN IN BOLD

Problem	HREA	MMEAWI	MO_Ring_PSO_SCD	TriMOEATAR	MMOEADC	DNNSGAII	HDMMODE
HDMMF1	7.0139e+2 (7.85e+0) +	3.4400e+2 (3.04e+0) +	1.3058e+2 (3.24e+0) +	2.6347e+1 (6.93e-1) -	1.1928e+2 (8.47e+0) +	**1.3599e+1 (2.11e-1) -**	1.0942e+2 (1.18e+1)
HDMMF2	1.5474e+2 (1.64e+1) +	1.1816e+2 (2.57e+0) +	1.3507e+2 (4.67e+0) +	**1.3078e+1 (1.50e+0) -**	7.9170e+1 (3.54e+0) =	1.4925e+1 (4.87e-1) -	8.0990e+1 (1.46e+1)
HDMMF3	3.8713e+1 (2.39e+0) -	1.2995e+2 (3.82e+0) +	1.3644e+2 (5.18e+0) +	**1.4286e+1 (2.31e+0) -**	7.4094e+1 (2.32e+0) -	1.6346e+1 (1.53e+0) -	9.9013e+1 (3.46e+0)
HDMMF4	9.0280e+1 (7.16e+0) +	1.0368e+2 (5.55e+0) +	1.4108e+2 (7.09e+0) +	**1.1873e+1 (2.13e+0) -**	6.9390e+1 (2.51e+0) -	1.7692e+1 (1.92e+0) -	7.6041e+1 (5.90e+0)
HDMMF5	5.3040e+1 (2.14e+1) -	9.7558e+1 (1.68e+0) =	1.5503e+2 (5.40e+0) +	**1.0883e+1 (1.32e+0) -**	1.2287e+3 (2.03e+2) +	1.9004e+1 (1.44e+0) -	9.2602e+1 (1.03e+1)
HDMMF6	5.0162e+1 (2.43e+1) -	9.5270e+1 (2.62e+0) +	1.5202e+2 (6.11e+0) +	**1.1387e+1 (2.18e+0) -**	6.2091e+1 (1.86e+0) -	1.8206e+1 (1.80e+0) -	8.7656e+1 (5.84e+0)
HDMMF7	6.2444e+1 (7.70e+0) -	1.0539e+2 (5.53e+0) +	1.6206e+2 (9.42e+0) +	5.8389e+1 (4.27e+1) =	7.4626e+1 (1.55e+0) -	**2.0575e+1 (2.06e+0) -**	8.1912e+1 (4.46e+0)
HDMMF8	4.1503e+1 (1.90e+1) -	1.0022e+2 (4.26e+0) +	1.4886e+2 (5.32e+0) +	**1.0270e+1 (1.42e+0) -**	6.8282e+1 (1.25e+0) -	1.7132e+1 (1.58e+0) -	9.0017e+1 (6.21e+0)
HDMMF9	1.8463e+3 (5.12e+1) +	2.7122e+2 (4.75e+0) +	1.6903e+2 (5.27e+0) +	4.1723e+1 (2.75e+0) -	5.2297e+2 (1.44e+2) +	**1.8374e+1 (2.50e+0) -**	1.0787e+2 (7.01e+0)
HDMMF10	1.5049e+2 (1.13e+1) +	1.6075e+2 (5.25e+0) =	1.4894e+2 (5.29e+0) -	2.8777e+1 (2.42e+0) -	8.6502e+1 (3.63e+0) -	**1.7990e+1 (2.30e+0) -**	1.6133e+2 (2.88e+1)
HDMMF11	4.5213e+2 (9.79e+0) -	2.9102e+2 (6.47e+0) +	1.5160e+2 (6.08e+0) -	**1.7691e+1 (2.57e+0) -**	7.0174e+2 (7.65e+1) +	1.9932e+1 (2.85e+0) -	2.3167e+2 (3.99e+1)
HDMMF12	2.2531e+2 (6.61e+0) +	1.7009e+2 (4.63e+0) +	1.4929e+2 (4.93e+0) +	2.3631e+1 (1.79e+0) -	9.8464e+1 (3.93e+0) -	**1.7432e+1 (2.13e+0) -**	1.4494e+2 (6.66e+0)
HDMMF13	1.4347e+2 (6.54e+0) -	3.1165e+2 (9.30e+0) +	2.1665e+2 (8.80e+0) -	3.9283e+1 (6.48e+0) -	4.8524e+2 (4.97e+1) +	**2.5721e+1 (3.80e+0) -**	2.8023e+2 (1.68e+1)
HDMMF14	6.3949e+2 (1.38e+1) +	6.1190e+2 (5.43e+0) +	1.3589e+2 (4.37e+0) -	2.9768e+1 (8.02e-1) -	6.9759e+1 (2.33e+0) -	**1.3951e+1 (6.95e-1) -**	1.8859e+2 (1.04e+1)
HDMMF15	2.4464e+3 (7.92e+1) +	2.8777e+2 (8.64e+0) +	1.7828e+2 (1.07e+1) -	3.4584e+1 (3.76e+0) -	9.9153e+1 (3.79e+0) -	**2.0115e+1 (2.69e+0) -**	1.9068e+2 (1.52e+1)
+/-/=	8/7/0	13/0/2	10/5/0	0/14/1	5/9/1	0/15/0	

S-IV Fifteen Instantiations of HDMMFs

HDMMF1

$$\begin{cases} f_1 = x_1 \\ f_2 = g \cdot h \end{cases}$$

$$g = 1 + \sum_{i=2}^{D} (10 + \frac{y_{1,i}}{y_{2,i}})$$

$$y_{1,i} = 50 \cdot \sin\left(\sqrt{100(x_{i+1} - x_i)^2 + (x_i - x_{i-1})^2}\right)^2 + 100 r(x_2) - 10$$

$$y_{2,i} = 1 + ((x_{i+1} - x_i)^2 - 2(x_{i+1} - x_i)(x_i - x_{i-1}) + (x_i - x_{i-1})^2)^2$$

$$r = \begin{cases} 25(x-6)^2, x > 5 \\ \left|0.508333333333333x^3 - 1.641666666666664x^2 + 2.5\right|, 0 \le x \le 5 \\ 1.5(x+1)^2 + 1, x < 0 \end{cases}$$

$$h = 1 - (\frac{x_1}{g})^2 - (\frac{x_1}{g}) \cdot \sin(2\pi \cdot q \cdot x_1)$$

Its search space is:

$x_1 \in [0,1]$, $x_j \in [-10,10], j = 2...,D$

Its PSs are:

$x_1 \in [0,1]$, $x_{2,...,D} = 2$、 6、 -7

Its PFs are:

$$f_2 = 1 - f_1^2 - f_1 \cdot \sin(8\pi \cdot f_1)$$

where $0 \le f_1, f_2 \le 2$

HDMMF2

$$\begin{cases} f_1 = x_1 \\ f_2 = \dfrac{g(t)}{x_1} \end{cases}$$

$$g(t) = 1 + 10 \cdot (1 - \sin(\frac{n}{8}\pi \cdot (t_1 - 2)))^2 + 5 \cdot \sum_{i=2}^{d} i(2t_i - t_{i-1})^2, t_i = \sum_{j=2}^{i} x_j, \quad i = 2, ..., D$$

Its search space is:

$x_1 \in [0.1, 1.1]$, $\quad x_j \in [-5, 5], j = 2..., D$

Its PSs are:

$$x_1 \in [0.1, 1.1], \quad x_j = \begin{cases} -4 \cdot (\frac{1}{2})^{j-2}, x_2 = 4, j = 3..., D, \\ 4 \cdot (\frac{1}{2})^{j-2}, x_2 = -4, j = 3..., D \end{cases}$$

Its PFs are:

$$f_1 \cdot f_2 = 1$$

where $0 \leq f_1, f_2 \leq 10$

HDMMF3

$$d = [0.15D], D \geq 10$$

$$\begin{cases} f_1 = g(\theta_1)\left[1 + W_1 \cdot p(x_1, x_2)^2 + W_2 \cdot q(x_3, \cdots, x_{2+d})^2 + W_3 \cdot h(x_{3+d}, \cdots, x_D)^2\right] \\ f_2 = g(\theta_1)\left[1 + W_1 \cdot p(x_1, x_2)^2 + W_2 \cdot q(x_3, \cdots, x_{2+d})^2 + W_3 \cdot h(x_{3+d}, \cdots, x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta_1) \\ g_2(\theta) = \sin(\frac{\pi}{2}\theta_1) \end{cases}$$

$$\theta = \begin{cases} \frac{2}{\pi}\arctan\left(\frac{|x_2|}{x_1}\right), \ x_1 \in [0,1] \\ \frac{2}{\pi}\arctan\left(\frac{|x_2|}{x_1+1}\right), \ x_1 \in [-1,0) \end{cases}$$

$$p(x_1, x_2) = \begin{cases} 0.97 - \sum_{i=1}^{2} x_i^2 \\ 0.25 - 6x_2^2 - (x_1+1)^2 \end{cases}$$

$$q(x_3, \cdots, x_{2+d}) = 50 \cdot (1 - \sin(\frac{2}{4}\pi \cdot t_i)^5), t_i = \sum_{j=3}^{2+i} x_j$$

$$h(x_{2+d+1}, \cdots, x_D) = -20\exp(-0.2\frac{\sqrt{(\sum_{i=3+d}^{D}(x_i - 0.2))^2}}{D-2-d}) - \exp(\frac{\sum_{i=3+d}^{D}\cos(2\pi(x_i - 0.2))}{D-2-d}) + 20 + e$$

Its search space is:

$x_1 \in [-1,1]$, $x_j \in [0,6], j = 2..., D$

Its PSs are:

$$\begin{cases} x_1 + x_2 = 0.97 \\ (x_1+1)^2 + 6x_2^2 = 0.25 \end{cases}, \quad \sum_{j=3}^{2+d} x_j = 1{,}5, j = 3..., 2+d, \quad x_j = 0, j = d+3, ..., D$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \leq f_1, f_2 \leq 1$

HDMMF4

$$m \geq 2, d \geq 7, D \geq 10$$

$$\begin{cases} f_1 = g(\theta_1)\left[1 + W_1 \cdot p(x_1, \cdots, x_m)^2 + W_2 \cdot q(x_{m+1}, \cdots, x_{m+d})^2 + W_3 \cdot h(x_{m+d+1}, \cdots, x_D)^2\right] \\ \cdots \\ f_m = g(\theta_m)\left[1 + W_1 \cdot p(x_1, \cdots, x_m)^2 + W_2 \cdot q(x_{m+1}, \cdots, x_{m+d})^2 + W_3 \cdot h(x_{m+d+1}, \cdots, x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = 1 - \cos(\frac{\pi}{2}\theta_1) \\ g_2(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\cos(\frac{\pi}{2}\theta_2) \\ \ldots \\ g_{m-1}(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\ldots\sin(\frac{\pi}{2}\theta_{m-2})\cos(\frac{\pi}{2}\theta_{m-1}) \\ g_m(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\ldots\sin(\frac{\pi}{2}\theta_{m-1}) \end{cases}$$

$$p(x_1, \cdots, x_m) = 1 - \sum_{i=1}^{m} x_i^2$$

$$q(x_{m+1}, \cdots, x_{m+d}) = 100 \cdot (1 - \sin(\frac{2}{8}\pi(t_1 - 2)))^2 + 50 \cdot \sum_{i=2}^{d} i(0.96t_i + t_{i-1})^2 + 20 \cdot p(x_1, \cdots, x_m)^2$$

$$t_i = \sum_{j=m+1}^{i} x_j, i = m+1, \ldots, m+d$$

$$h(x_{m+d+1}, \cdots, x_D) = \sum_{i=m+d+1}^{D} (x_i - 0.2)^2$$

Its search space is:

$$x_i \in [0,1], i = 1, \ldots, m, \quad x_j \in [-5,5], j = m+1, \ldots, d$$

Its PSs are:

$$\sum_{i=1}^{m} x_i^2 = 1$$,

$$x_{m+1} = \begin{cases} 4.5, \\ 2.5 \\ 0.5 \\ -1.5 \\ -3.5 \end{cases}, \quad x_{m+2} = x_{m+1} \cdot \frac{1-\alpha}{\alpha}, \quad x_j = x_{m+2} \cdot (\frac{1}{\alpha})^{j-m-2}, j = m+3 \ldots, D-1, \ x_D = 0.2$$

Its PFs are:

$$(1-f_1)^2+(1-f_2)^2=1$$

where $0 \le f_1, f_2 \le 1$

HDMMF5

$m \ge 2, k=3, d \ge 12, D \ge 10$

$$
\begin{cases}
f_1 = g(\theta_1)\left[1 + W_1 \cdot p(x_1, \cdots, x_m)^2 + W_2 \cdot q(x_{m+1}, \cdots, x_{m+d})^2 + W_3 \cdot h(x_{m+d+1}, \cdots, x_D)^2\right] \\
\cdots \\
f_m = g(\theta_m)\left[1 + W_1 \cdot p(x_1, \cdots, x_m)^2 + W_2 \cdot q(x_{m+1}, \cdots, x_{m+d})^2 + W_3 \cdot h(x_{m+d+1}, \cdots, x_D)^2\right]
\end{cases}
$$

$$
\begin{cases}
g_1(\theta) = 1 - \cos(\frac{\pi}{2}\theta_1) \\
g_2(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\cos(\frac{\pi}{2}\theta_2) \\
\cdots \\
g_{m-1}(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)...\sin(\frac{\pi}{2}\theta_{m-2})\cos(\frac{\pi}{2}\theta_{m-1}) \\
g_m(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)...\sin(\frac{\pi}{2}\theta_{m-1})
\end{cases}
$$

$$p(x_1, \cdots, x_m) = 1 - \sum_{i=1}^{m} x_i^2$$

$$q(x_{m+1}, \cdots, x_{m+d}) = \sum_{j=1}^{13} (t_3 e^{-0.1 i \cdot t_1} - t_4 e^{-0.1 i \cdot t_2} + t_6 e^{-0.1 i \cdot t_5} - e^{-0.1 i} + 5e^{i} - 3e^{-0.4 i})^2$$

$$t_1 = 100((x_{i+1} - 3) - 0.01x_i^2 + 1)^2 + 0.01r_i + 1$$

$$t_2 = 100((x_{i+3} - 3) - 0.01x_{i+2}^2 + 1)^2 + 0.01r_{i+2} + 10$$

$$t_3 = 100((x_{i+5} - 3) - 0.01x_{i+4}^2 + 1)^2 + 0.01(x_{i+4} + 10)^2 + 1$$

$$t_4 = 100((x_{i+7} - 3) - 0.01x_{i+6}^2 + 1)^2 + 0.01(x_{i+6} + 10)^2 + 5$$

$$t_5 = 100((x_{i+9} - 3) - 0.01x_{i+8}^2 + 1)^2 + 0.01(x_{i+8} + 10)^2 + 4$$

$$t_6 = 100((x_{i+11} - 3) - 0.01x_{i+10}^2 + 1)^2 + 0.01(x_{i+10} + 10)^2 + 3$$

$$
r = \begin{cases}
25(x-6)^2, x > 5 \\
(x+10)^2, x < -5 \\
\frac{24}{25}x^2 + 1, -5 \le x \le 5
\end{cases}
$$

$$h(x_{m+d+1}, \cdots, x_D) = \sum_{i=m+d+1}^{D-1} (x_i^2 + 2x_{i-1}^2 - 0.3\cos(3\pi(x_i - 1)) \cdot \cos(4\pi(x_{i+1} - 1)) + 0.3)$$

Its search space is:

$x_i \in [0,15], i = 1,2,...,m, \qquad x_j \in [-15,15], j = m+1...,\mathrm{d}$

Its PSs are:

$$\sum_{i=1}^{m} x_i^2 = 1,$$

Its PFs are:

$$(1-f_1)^2+(1-f_2)^2=1$$

where $0\le f_1, f_2\le 1$

NMMF6

$m\ge 2, k=3, d\ge 8, D\ge 12$

$$\begin{cases} f_1 = g(\theta_1)\left[1+W_1\cdot p(x_1,\cdots,x_m)^2 + W_2\cdot q(x_{m+1},\cdots,x_{m+d})^2 + W_3\cdot h(x_{m+d+1},\cdots,x_D)^2\right] \\ \cdots \\ f_m = g(\theta_m)\left[1+W_1\cdot p(x_1,\cdots,x_m)^2 + W_2\cdot q(x_{m+1},\cdots,x_{m+d})^2 + W_3\cdot h(x_{m+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta_1) \\ g_2(\theta) = \sin(\frac{\pi}{2}\theta_1)\cos(\frac{\pi}{2}\theta_2) \\ \cdots \\ g_{m-1}(\theta) = \sin(\frac{\pi}{2}\theta_1)\ldots\sin(\frac{\pi}{2}\theta_{m-2})\cos(\frac{\pi}{2}\theta_{m-1}) \\ g_m(\theta) = \sin(\frac{\pi}{2}\theta_1)\ldots\sin(\frac{\pi}{2}\theta_{m-1}) \end{cases}$$

$$p(x_1,\cdots,x_m) = 1-\sum_{i=1}^{m} x_i^2$$

$$q(x_{m+1},\cdots,x_{m+d}) = (e^{-t_1}-t_2)^4 + 100(t_2-t_3)^6 + \tan(5000(t_3-t_4))^4 + t_1^8$$

$$t_1 = 100((x_{i+1}-3)-0.01x_i^2+1)^2 + 0.01r_i$$

$$t_2 = 100((x_{i+3}-3)-0.01x_{i+2}^2+1)^2 + 0.01r_{i+2} + 1$$

$$t_3 = -100((x_{i+5}-3)-0.01x_{i+4}^2+1)^2 + 0.01(x_{i+4}+10)^2 + 1$$

$$t_4 = -100((x_{i+7}-3)-0.01x_{i+6}^2+1)^2 + 0.01(x_{i+6}+10)^2 + 1$$

$$r = \begin{cases} 25(x-6)^2, x>5 \\ (x+10)^2, x<-5 \\ \frac{24}{25}x^2+1, -5\le x\le 5 \end{cases}$$

$$h(x_{m+d+1},\cdots,x_D) = \sum_{i=m+d+1}^{D-1}(x_i^2+2x_{i-1}^2-0.3\cos(3\pi(x_i-1))\cdot\cos(4\pi(x_{i+1}-1))+0.3)$$

Its search space is:

$x_i \in [-1,1], i=1,2,...,m$, $x_j \in [-15,15], j=m+1...,\mathrm{d}$

Its PSs are:

$$\sum_{i=1}^{m} x_i^2 = 1$$

$$x_j = \begin{cases} -10, j=m+1, m+3 \\ 6, j=m+1, m+3 \end{cases}, \qquad x_j = \begin{cases} 3, j=m+2, m+4 \\ 2.36, j=m+2, m+4 \end{cases},$$

$$\begin{cases} x_j = -10, j = m+5,...,m+11 \\ x_l = 3, j = m+6,...,m+12 \end{cases}$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \le f_1, f_2 \le 1$

NMMF7

$$m \ge 2, k = 3, d \ge 8, D \ge 12$$

$$\begin{cases} f_1 = g(\theta_1)\left[1 + W_1 \cdot p(x_1,\cdots,x_m)^2 + W_2 \cdot q(x_{m+1},\cdots,x_{m+d})^2 + W_3 \cdot h(x_{m+d+1},\cdots,x_D)^2\right] \\ \cdots \\ f_m = g(\theta_m)\left[1 + W_1 \cdot p(x_1,\cdots,x_m)^2 + W_2 \cdot q(x_{m+1},\cdots,x_{m+d})^2 + W_3 \cdot h(x_{m+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta_1) \\ g_2(\theta) = \sin(\frac{\pi}{2}\theta_1)\cos(\frac{\pi}{2}\theta_2) \\ \cdots \\ g_{m-1}(\theta) = \sin(\frac{\pi}{2}\theta_1)...\sin(\frac{\pi}{2}\theta_{m-2})\cos(\frac{\pi}{2}\theta_{m-1}) \\ g_m(\theta) = \sin(\frac{\pi}{2}\theta_1)...\sin(\frac{\pi}{2}\theta_{m-1}) \end{cases}$$

$$p(x_1,\cdots,x_m) = 1 - \sum_{i=1}^{m} x_i^2$$

$$q(x_{m+1},\cdots,x_{m+d}) = (e^{-t_1} - t_2)^4 + 100(t_2 - t_3)^6 + \tan(5000(t_3 - t_4))^4 + t_1^8$$

$$t_1 = 100((x_{i+1} - 3) - 0.01x_i^2 + 1)^2 + 0.01r_i + 20p(x_1,\cdots,x_m)^2$$

$$t_2 = 100((x_{i+3} - 3) - 0.01x_{i+2}^2 + 1)^2 + 0.01r_{i+2} + 1 + 20p(x_1,\cdots,x_m)^2$$

$$t_3 = -100((x_{i+5} - 3) - 0.01x_{i+4}^2 + 1)^2 + 0.01(x_{i+4} + 10)^2 + 1 + 20p(x_1,\cdots,x_m)^2$$

$$t_4 = -100((x_{i+7} - 3) - 0.01x_{i+6}^2 + 1)^2 + 0.01(x_{i+6} + 10)^2 + 1 + 20p(x_1,\cdots,x_m)^2$$

$$h(x_{m+d+1},\cdots,x_D) = \sum_{i=m+d+1}^{D-1} (x_i^2 + 2x_{i-1}^2 - 0.3\cos(3\pi(x_i - 1)) \cdot \cos(4\pi(x_{i+1} - 1)) + 0.3)$$

Its search space is:

$x_i \in [-1,1], i = 1,2,...,m$, $x_j \in [-15,15], j = m+1...,\mathrm{d}$

Its PSs are:

$$\sum_{i=1}^{m} x_i^2 = 1$$

$$x_j = \begin{cases} -10, j = m+1, m+3 \\ 6, j = m+1, m+3 \end{cases}, x_j = \begin{cases} 3, j = m+2, m+4 \\ 2.36, j = m+2, m+4 \end{cases}, \quad \begin{cases} x_j = -10, j = m+5, ..., m+11 \\ x_l = 3, j = m+6, ..., m+12 \end{cases}$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \le f_1, f_2 \le 1$

NMMF8

$$m \ge 2, k = 3, d \ge 8, dd = 2, D \ge 12$$

$$\begin{cases} f_1 = g(\theta_1)\left[1 + W_1 \cdot p(x_1,\cdots,x_m)^2 + W_2 \cdot q(x_{m+1},\cdots,x_{m+d})^2 + W_3 \cdot h(x_{m+d+1},\cdots,x_D)^2\right] \\ \cdots \\ f_m = g(\theta_m)\left[1 + W_1 \cdot p(x_1,\cdots,x_m)^2 + W_2 \cdot q(x_{m+1},\cdots,x_{m+d})^2 + W_3 \cdot h(x_{m+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = 1 - \cos(\frac{\pi}{2}\theta_1) \\ g_2(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\cos(\frac{\pi}{2}\theta_2) \\ \cdots \\ g_{m-1}(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\ldots\sin(\frac{\pi}{2}\theta_{m-2})\cos(\frac{\pi}{2}\theta_{m-1}) \\ g_m(\theta) = 1 - \sin(\frac{\pi}{2}\theta_1)\ldots\sin(\frac{\pi}{2}\theta_{m-1}) \end{cases}$$

$$p(x_1,\cdots,x_m) = 1 - \sum_{i=1}^{m} x_i^2$$

$$t_1 = \frac{\left|\sum_{i=m+1}^{m+dd} x_i\right|}{6}, t_2 = \frac{\left|\sum_{i=m+dd+1}^{m+2dd} x_i\right|}{3}, t_3 = \frac{\left|\sum_{i=m+2dd+1}^{m+3dd} x_i\right|}{2}$$

$$a = 2t_1^3 + 5t_1t_2 + 4t_3 - 2t_1^2t_3 - 18$$

$$b = t_1 + t_2^3 + t_1 \cdot t_3^2 - 18$$

$$c = 8t_1^2 + 2t_2t_3 + 2t_2^2 + 3t_3^3 - 52$$

$$q(x_{m+1},\cdots,x_{m+3dd}) = (ab^2c + abc^2 + b^2 + (t_1 + t_2 - t_3)^2 + 20(1 - x_1^2 - x_2^2)^2$$

$$h(x_{m+d+1},\ldots,x_D) = -20\exp(-0.2\frac{\sqrt{(\sum_{i=m+d+1}^{D}(x_i - 0.2))^2}}{D-m-d}) - \exp(\frac{\sum_{i=m+d+1}^{D}\cos(2\pi(x_i - 0.2))}{D-m-d}) + 20 + e$$

Its search space is:

$x_i \in [0,1], i = 1,2,\ldots,m$, $x_j \in [-6,6], j = m+1\ldots,\mathrm{D}$

Its PSs are:

$$\sum_{i=1}^{m} x_i^2 = 1$$

,

$$\begin{cases} x_{m+1} + x_{m+dd} = 6, x_{m+dd+1} + x_{m+2dd} = 6 \\ x_{m+1} + x_{m+dd} = -6, x_{m+dd+1} + x_{m+2dd} = 6 \\ x_{m+1} + x_{m+dd} = 6, x_{m+dd+1} + x_{m+2dd} = -6 \\ x_{m+1} + x_{m+dd} = -6, x_{m+dd+1} + x_{m+2dd} = -6 \end{cases}, , x_{m+2dd+1} + x_{m+3dd} = 6, x_{m+3dd+1},\ldots,x_D = 0.2$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \le f_1, f_2 \le 1$

NMMF9

$$m \ge 2, k = 3, q_1 \ge 8, q_2 \ge 8, D \ge 25$$
$$d = q_1 + q_2$$

$$\begin{cases} f_1 = g(\theta)\left[1+W_1\cdot p(x_1,\cdots,x_k)^2+W_2\cdot q(x_{k+1},\cdots,x_{k+d})^2+W_3\cdot h(x_{k+d+1},\cdots,x_D)^2\right] \\ f_2 = g(\theta)\left[1+W_1\cdot p(x_1,\cdots,x_k)^2+W_2\cdot q(x_{k+1},\cdots,x_{k+d})^2+W_3\cdot h(x_{k+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\theta = \frac{2}{\pi}\arctan\left(\frac{\sqrt{{x_2}^2+x_1^2}}{x_3}\right)$$

$$p(x_1,\cdots,x_k) = 1-\sum_{i=1}^{k} x_i^2$$

$$q_1(x_{k+1,\dots,}x_{k+q_1}) = 2-\exp(-2\log_2(2-\sin(2\pi(\sum_{i=k+1}^{k+q_1} x_i-\frac{1}{4}))) \cdot \sin^6(\pi\sum_{i=k+1}^{k+q_1} x_i)-1$$

$$\text{for } l_1 \to \begin{cases} x_i^* = (1+\cos(\frac{\pi}{2}\cdot\frac{i}{k+q_1+q_2}))(x_i-L_i)-\theta\cdot(U_i-L_i), \quad k+q_1+1\le i\le k+q_1+q_2, i \text{ is } even. \\ x_i^* = (1+\frac{i}{k+q_1+q_2})(x_i-L_i)-\frac{x_4\text{-}L_4}{U_4-L_4}\cdot(U_i-L_i), \quad k+q_1+1\le i\le k+q_1+q_2, i \text{ is } odd \end{cases}$$

$$q_2(x_{k+q_1+1,\dots,}x_{k+q_1+q_2}) = \sum_{i=k+q_1+1}^{k+q_1+q_2-1}((x_{i-1}+10x_i)^2+5(x_{i+1}-x_{i+2})^2+(x_i-2x_{i+1})^4+10(x_{i-1}-x_{i+2})^4)$$

$$q(x_{k+1,\dots}x_{k+d}) = q_1(x_{k+1,\dots,}x_{k+q_1})+q_2(x_{k+q_1+1,\dots,}x_{k+q_1+q_2})$$

$$h(x_{k+d+1},\cdots,x_D) = -20\exp(-0.2\sqrt{\frac{(\sum_{i=k+d+1}^{D}(x_i-0.2))^2}{D-k-d}})-\exp(\frac{\sum_{i=k+d+1}^{D}\cos(2\pi(x_i-0.2))}{D-k-d})+20+e$$

Its search space is:

$$x_i\in[0,1], i=1,2,\dots,k,$$
$$x_i\in[0,5], i=k+1,\dots,k+d, ,$$
$$x_i\in[0,1], i=k+d+1,\dots,D,$$

Its PSs are:

$$\sum_{i=1}^{k} x_i^2 = 1 \qquad \sum_{j=k+1}^{k+q_1} x_j = \begin{cases} 0.5 \\ 2.5, j=1,\dots,k+q_1 \\ 4.5 \end{cases}$$

$$x_j = \begin{cases} \dfrac{\theta}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1})}, j=odd \\ \dfrac{x_{k+1}}{5(1+\frac{i}{k+q_1+q_2})}, j=even \end{cases}, j=k+q_1,\dots,k+q_1+q_2$$

$$x_j = \frac{x_{k+q_1+1}}{1+\cos(\frac{\pi}{2}\frac{j}{D})}, j = k + q_1 + q_2 + 1, \ldots D$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \le f_1, f_2 \le 1$

$m \ge 2, k = 3, d \ge 9, D \ge 15$

NMMF10

$$\begin{cases} f_1 = g_1(\theta)\left[1 + W_1 \cdot p(x_1, \cdots, x_k)^2 + W_2 \cdot q(x_{k+1}, \cdots, x_{k+d})^2 + W_3 \cdot h(x_{k+d+1}, \cdots, x_D)^2\right] \\ f_2 = g_2(\theta)\left[1 + W_1 \cdot p(x_1, \cdots, x_k)^2 + W_2 \cdot q(x_{k+1}, \cdots, x_{k+d})^2 + W_3 \cdot h(x_{k+d+1}, \cdots, x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta) \\ g_2(\theta) = \sin(\frac{\pi}{2}\theta)\cos(\frac{\pi}{2}\theta) \end{cases}$$

$$\theta = \frac{2}{\pi}\arctan\left(\frac{\sqrt{{x_2}^2 + x_3^2}}{x_1}\right)$$

$$p(x_1, \cdots, x_k) = 1 - \sum_{i=1}^{k} x_i^2$$

$$x_{k+1} = (1 + \cos(\frac{\pi}{2} \cdot \frac{1}{2}))(x_i - L_i) - x_1 \cdot (U_i - L_i), \quad i = k+1$$

$$t = \sum_{i=k+1}^{k+d-1} x_i$$

$$q(x_{k+1} \quad x_{k+d}) = 100(1 - \sin(\frac{\pi}{4} \sum_{i=k+1}^{k+d-1} x_i - \quad)^{\;\;2} + \quad \sum_{i=k+2}^{k+d-1} i \cdot \quad \sum_{j_1=k+2}^{k+d-1} x_{j_1} + \sum_{j_2=k+1}^{k+d-2} x_{j_2})^2$$
$$+ 50(k+d)(2x_{k+d} - x_1 - x_2)^2$$

$$h(x_{k+d+1}, \ldots x_D) = \sum_{i=k+d+1}^{D} (x_i - 0.2)^2$$

Its search space is:

$$x_i \in [0,1], i = 1, 2, \ldots, k$$
$$x_i \in [-10,10], i = k+1, \ldots, k+d \;,$$
$$x_i \in [0,1], i = k+d+1, \ldots, D$$

Its PSs are:

$$\sum_{i=1}^{k} x_i^2 = 1, x_{k+1} = \begin{cases} \frac{2}{3} \cdot \frac{20x_1 + 4}{1 + \sqrt{0.5}/2 - 10}, \\ \frac{2}{3} \cdot \frac{20x_1 - 4}{1 + \sqrt{0.5}/2 - 10} \end{cases}$$

$$x_{k+2} = -6, x_{k+2+i} = x_{k+2} \cdot (-\frac{1}{2})^i, i = 1, \ldots, d-1 \;, \;,$$

$$x_{k+d} = \frac{x_1 + x_2}{2}, x_{k+d+1}, \ldots, x_D = 0.2$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \le f_1, f_2 \le 1$

$k = 3, q_1 = 5, q_2 = 5, q_3 = 5, D \ge 18$

NMMF11

$$\begin{cases} f_1 = g_1(\theta)\left[1 + W_1 \cdot p(x_1,\cdots,x_k)^2 + W_2 \cdot q(x_{k+1},\cdots,x_{k+d})^2 + W_3 \cdot h(x_{k+d+1},\cdots,x_D)^2\right] \\ f_2 = g_2(\theta)\left[1 + W_1 \cdot p(x_1,\cdots,x_k)^2 + W_2 \cdot q(x_{k+1},\cdots,x_{k+d})^2 + W_3 \cdot h(x_{k+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta) \\ g_2(\theta) = \sin(\frac{\pi}{2}\theta)\cos(\frac{\pi}{2}\theta) \end{cases}$$

$$\theta = \frac{2}{\pi}\arctan\left(\frac{\sqrt{x_2{}^2 + x_1^2}}{x_3}\right)$$

$$p(x_1,\cdots,x_k) = 1 - \sum_{i=1}^{k} x_i^2$$

$$\text{for } l_1 \rightarrow \begin{cases} x_i^* = \quad + \quad \frac{\pi}{2} \cdot \frac{i}{k+q_1}) \quad x_i - L_i \ -\theta \cdot \ U_i - L_i \quad k + \ \le i \le k + q \quad i \quad even \\ x_i^* = (1 + \frac{i}{k+q_1})(2x_i - L_i) - x_1 \cdot (U_i - L_i), \quad k+1 \le i \le k + q_1,\ i \text{ is } odd \end{cases}$$

$$\text{for } l_2 \rightarrow x_i^* = (1 + \cos(\frac{\pi}{2} \cdot \frac{i}{k+q_1+q_2}))(2x_i - L_i) - \frac{x_{k+1} - L_{k+1}}{U_{k+1} - L_{k+1}} \cdot (U_i - L_i), \quad k + q_1 + 1 \le i \le k + q_1 + q_2$$

$$\text{for } l_3 \rightarrow \begin{cases} x_i^* = (1 + \cos(\frac{\pi}{2} \cdot \frac{i}{k+q_1+q_2+q_3}))(2x_i - L_i) - \theta \cdot (U_i - L_i), \quad k + q_1 + 1 \le i \le k + q_1 + q_2,\ i \text{ is } even. \\ x_i^* = (1 + \frac{i}{k+q_1+q_2+q_3})(2x_i - L_i) - x_1 \cdot (U_i - L_i), \quad k + q_1 + 1 \le i \le k + q_1 + q_2,\ i \text{ is } odd \end{cases}$$

$$q(x_{k+1},\ldots x_{k+q_1}) = \sum_{i=k+1}^{k+q_1-1} ((|x_i| - \frac{k+i}{k+q_1})^2 + 100(x_{i+1} - \frac{k+i+1}{k+i}x_i)^2)^{1/4} \cdot (\sin^2(50((|x_i| - \frac{k+i}{k+q_1})^2 + 10(x_{i+1} - \frac{k+i+1}{k+i}x_i)^2)^{1/10}) + 100)$$

$$y = \sum_{i=k+q_1+1}^{k+q_1+q_2} \cos(\frac{(x_i+10)\pi}{20})$$

$$q(x_{k+q_1+1},\ldots x_{k+q_1+q_2}) = \sum_{i=k+q_1+1}^{k+q_1+q_2} (q_2 - y + i \cdot (1 - \cos(x_i) - \sin(x_i)))^2$$

$$q(x_{k+q_1+q_2+1},\ldots x_{k+q_1+q_2+q_3}) = \sum_{i=k+q_1+q_2+1}^{k+q_1+q_2+q_3-1} ((|x_i| - \frac{k+i}{k+q_1})^2 + 100(x_{i+1} - \frac{k+i+1}{k+i}x_i)^2)^{1/4} \cdot (\sin^2(50((|x_i| - \frac{k+i}{k+q_1})^2 + 10(x_{i+1} - \frac{k+i+1}{k+i}x_i$$

$$h(x_{k+q_1+q_2+q_3},\ldots x_D) = \sum_{i=k+q_1+q_2+q_3}^{D} (x_i - 0.2)^2$$

Its search space is:

$$x_i \in [0,1], i = 1,2,\ldots,k, \qquad x_j \in [-10,10], j = k+1\ldots,D$$

Its PS1 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} (\dfrac{20\theta + \frac{i}{k+q_1}}{1 + \cos(\frac{\pi}{2}\frac{i}{k+q_1})} - 10)/2, j = odd \\ (\dfrac{20x_1 + \frac{i}{k+q_1}}{1 + \frac{i}{k+q_1}} - 10)/2, j = even \end{cases}, j = k+1,\ldots,k+q_1$$

$$x_j = \begin{cases} \left(\dfrac{x_{k+1}}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3}\right)}-10\right)/2, j = odd \end{cases}, j = k+q_1+1,\ldots,k+q_1+q_2$$

$$x_j = \begin{cases} \left(\dfrac{20\theta+\dfrac{i}{k+q_1+q_2+q_3}}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3}\right)}-10\right)/2, j = odd \\ \left(\dfrac{20x_1+\dfrac{i}{k+q_1+q_2+q_3}}{1+\dfrac{i}{k+q_1+q_2+q_3}}-10\right)/2, j = even \end{cases}, j = k+q_1+q_2+1,\ldots,k+q_1+q_2+q_3$$

$$x_{k+q_1+q_2+q_3+1},\ldots,x_D = 0.2$$

Its PS2 is:

$$x_j = \begin{cases} \left(\dfrac{20\theta-\dfrac{i}{k+q_1}}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1}\right)}-10\right)/2, j = odd \\ \left(\dfrac{20x_1-\dfrac{i}{k+q_1}}{1+\dfrac{i}{k+q_1}}-10\right)/2, j = even \end{cases}, j = k+1,\ldots,k+q_1$$

$$x_j = \begin{cases} \left(\dfrac{x_{k+1}}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3}\right)}-10\right)/2, j = odd \end{cases}, j = k+q_1+1,\ldots,k+q_1+q_2$$

$$x_j = \begin{cases} \left(\dfrac{20\theta+\dfrac{i}{k+q_1+q_2+q_3}}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3}\right)}-10\right)/2, j = odd \\ \left(\dfrac{20x_1+\dfrac{i}{k+q_1+q_2+q_3}}{1+\dfrac{i}{k+q_1+q_2+q_3}}-10\right)/2, j = even \end{cases}, j = k+q_1+q_2+1,\ldots,k+q_1+q_2+q_3$$

Its PS3 is:

$$x_j = \begin{cases} \left(\dfrac{20\theta - \dfrac{i}{k+q_1}}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1})} - 10\right)/2, j = odd \\ \left(\dfrac{20x_1 - \dfrac{i}{k+q_1}}{1+\dfrac{i}{k+q_1}} - 10\right)/2, j = even \end{cases}, j = k+1, ..., k+q_1$$

$$x_j = \left\{ \left(\dfrac{x_{k+1}}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3})} - 10\right)/2, j = c \right.$$

$$x_j = \begin{cases} \left(\dfrac{20\theta - \dfrac{i}{k+q_1+q_2+q_3}}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3})} - 10\right)/2, j = odd \\ \left(\dfrac{20x_1 - \dfrac{i}{k+q_1+q_2+q_3}}{1+\dfrac{i}{k+q_1+q_2+q_3}} - 10\right)/2, j = even \end{cases}, j = k+q_1+q_2+1, ..., k+q_1+q_2+q_3$$

Its PS4 is:

$$x_j = \begin{cases} \left(\dfrac{20\theta + \dfrac{i}{k+q_1}}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1})} - 10\right)/2, j = odd \\ \left(\dfrac{20x_1 + \dfrac{i}{k+q_1}}{1+\dfrac{i}{k+q_1}} - 10\right)/2, j = even \end{cases}, j = k+1, ..., k+q_1$$

$$x_j = \left\{ \left(\dfrac{x_{k+1}}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3})} - 10\right)/2, j = c \right.$$

$$x_j = \begin{cases} \left(\dfrac{20\theta - \dfrac{i}{k+q_1+q_2+q_3}}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2+q_3})} - 10\right)/2, j = odd \\ \left(\dfrac{20x_1 - \dfrac{i}{k+q_1+q_2+q_3}}{1+\dfrac{i}{k+q_1+q_2+q_3}} - 10\right)/2, j = even \end{cases}, j = k+q_1+q_2+1,...,k+q_1+q_2+q_3$$

Its PFs are:

$$f_1^2 + f_2^2 = 1$$

where $0 \le f_1, f_2 \le 1$

NMMF12

$k = 3, q_1 = 6, q_2 = 6, D \ge 18$

$$\begin{cases} f_1 = g_1(\theta)\left[1 + W_1 \cdot p(x_1, \cdots, x_k)^2 + W_2 \cdot q(x_{k+1}, \cdots, x_{k+d})^2 + W_3 \cdot h(x_{k+d+1}, \cdots, x_D)^2\right] \\ f_2 = g_2(\theta)\left[1 + W_1 \cdot p(x_1, \cdots, x_k)^2 + W_2 \cdot q(x_{k+1}, \cdots, x_{k+d})^2 + W_3 \cdot h(x_{k+d+1}, \cdots, x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta) \\ g_2(\theta) = \sin(\frac{\pi}{2}\theta)\cos(\frac{\pi}{2}\theta) \end{cases}$$

$$\theta = \frac{2}{\pi}\arctan\left(\frac{\sqrt{x_2{}^2 + x_1^2}}{x_3}\right)$$

$$p(x_1, \cdots, x_k) = 1 - \sum_{i=1}^{k} x_i^2$$

$$\text{for } l_1 \to \begin{cases} x_i^* = \quad + \qquad \frac{\pi}{2} \cdot \frac{i}{k+q_1} \qquad x_i - L_i \;\; - \theta \cdot (U_i - L_i \qquad k + \;\; \leq i \leq \; k + q_1 \;\; i \qquad even \\ x_i^* = (1 + \frac{i}{k+q_1})(1.8x_i - L_i) - x_1 \cdot (U_i - L_i), \quad k+1 \leq i \leq \; k + q_1, \, i \text{ is } odd \end{cases}$$

$$\text{for } l_2 \to x_i^* = (1 + \cos(\frac{\pi}{2} \cdot \frac{i}{k+q_1+q_2}))(1.8x_i - L_i) - \frac{x_{k+1}\text{-}L_{k+1}}{U_{k+1} - L_{k+1}} \cdot (U_i - L_i), \quad k + q_1 + 1 \leq i \leq \; k + q_1 + \text{q}_2$$

$$q(x_{k+1}, \ldots x_{k+q_1}) = \begin{cases} \sum_{i=k+1}^{k+q_1} 10(x_{k+1} - 1)^2 + \sum_{i=k+1}^{k+q_1} i(x_i - x_{i-1})^2 + \sum_{i=k+1}^{k+q_1} 20i\sin^2 A + \sum_{i=k+1}^{k+q_1} i\log_{10}(1 + iB^2) \\ A = 10x_{i-1}\sin(x_i - x_{i-1}) + 5\sin(x_{i+1} - x_i) \\ B = ((x_i - x_{i-1})^2 - ix_i + ix_{i+1} - 5\cos(x_{i+1} - x_i) + 5) \end{cases}$$

$$q(x_{k+q_1+1}, \ldots x_{k+q_1+q_2}) = 1 + \sum_{i=2}^{D}(5 + \frac{y_{1,i}}{y_{2,i}})$$

$$y_{1,i} = 50 \cdot \sin\left(\sqrt{100(x_{i+1} - x_i)^2 + (x_i - x_{i-1})^2}\right)^2 + 10r - 5$$

$$y_{2,i} = 1 + ((x_{i+1} - x_i)^2 - 2(x_{i+1} - x_i)(x_i - x_{i-1}) + (x_i - x_{i-1})^2)^2$$

$$r = \begin{cases} 25(x-6)^2, x > 5 \\ \frac{24}{25}x^2 + 1, -5 \leq x \leq 5 \\ \frac{25}{4}(x+7)^2, x < -5 \end{cases}$$

$$h(x_{k+q_1+q_2}, \ldots x_D) = \sum_{i=k+q_1+q_2}^{D}(x_i - 0.2)^2$$

Its search space is:

$$x_i \in [0,1], i = 1,2,\ldots,k$$

$$x_i \in [-10,10], i = k+1,\ldots,k+q_1+q_2 \, ,$$

$$x_i \in [0,1], i = k+q_1+q_2+1,\ldots,D$$

Its PS1 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} \left(\dfrac{20\theta + 6}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1}\right)} - 10\right)/(1.8),\ j = odd \\ \left(\dfrac{20x_1 + 6}{1+\dfrac{i}{k+q_1}} - 10\right)/(1.8),\ j = even \end{cases},\ j = k+1,\dots,k+q_1 \quad x_j = \left(\dfrac{x_{k+1} + 16}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2}\right)} - 10\right)/(1.8),\ j =$$

$$x_{k+q_1+q_2+1},\dots,x_D = 0.2$$

Its PS2 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} \left(\dfrac{20\theta - 7}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1}\right)} - 10\right)/(1.8),\ j = odd \\ \left(\dfrac{20x_1 - 7}{1+\dfrac{i}{k+q_1}} - 10\right)/(1.8),\ j = even \end{cases},\ j = k+1,\dots,k+q_1 \quad x_j = \left(\dfrac{x_{k+1} + 16}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2}\right)} - 10\right)/(1.8),\ j =$$

$$x_{k+q_1+q_2+1},\dots,x_D = 0.2$$

Its PS3 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} \left(\dfrac{20\theta - 7}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1}\right)} - 10\right)/(1.8),\ j = odd \\ \left(\dfrac{20x_1 - 7}{1+\dfrac{i}{k+q_1}} - 10\right)/(1.8),\ j = even \end{cases},\ j = k+1,\dots,k+q_1 \quad x_j = \left(\dfrac{x_{k+1} + 3}{1+\cos\left(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2}\right)} - 10\right)/(1.8),\ j =$$

$$x_{k+q_1+q_2+1},\dots,x_D = 0.2$$

Its PS4 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} \left(\dfrac{20\theta+6}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1})} - 10\right)/(1.8), j = odd \\ \left(\dfrac{20x_1+6}{1+\frac{i}{k+q_1}} - 10\right)/(1.8), j = even \end{cases}, j = k+1,...,k+q_1 \quad x_j = \left(\frac{x_{k+1}+3}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})} - 10\right)/(1.8), j =$$

$$x_{k+q_1+q_2+1},...,x_D = 0.2$$

Its PFs are:

$$f_1 + f_2 = 1$$

where $0 \le f_1, f_2 \le 1$

NMMF13

$$m = 3, k = 3, dd_1 = 50, D \ge 60$$

$$\begin{cases} f_1 = g_1(\theta_1)\left[1+W_1\cdot p(x_1,\cdots,x_k)^2+W_2\cdot q(x_{k+1},\cdots,x_{k+d})^2+W_3\cdot h(x_{k+d+1},\cdots,x_D)^2\right] \\ \cdots \\ f_3 = g_3(\theta_2)\left[1+W_1\cdot p(x_1,\cdots,x_k)^2+W_2\cdot q(x_{k+1},\cdots,x_{k+d})^2+W_3\cdot h(x_{k+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta) = \cos(\frac{\pi}{2}\theta_1) \\ \cdots \\ g_3(\theta) = \sin(\frac{\pi}{2}\theta_2)\cos(\frac{\pi}{2}\theta_2) \end{cases}$$

$$\begin{cases} \theta_1 = \frac{2}{\pi}\arctan\left(\frac{\sqrt{x_2{}^2+x_3^2}}{x_1}\right) \\ \theta_2 = \frac{2}{\pi}\arctan\left(\frac{\sqrt{x_3{}^2}}{x_2}\right) \end{cases}$$

$$p(x_1,\cdots,x_k) = 1-\sum_{i=1}^{k} x_i^2$$

$$\text{for } l_1 \rightarrow \begin{cases} x_i^* = (1+\cos(\frac{\pi}{2}\cdot\frac{i}{k+d-1}))(1.5x_i - L_i) - x_1\cdot(U_i - L_i), \quad i = k+1,\ldots,k+d-1, i \text{ is } even \\ x_i^* = (1+\frac{i}{k+d-1})(1.5x_i - L_i) - x_2\cdot(U_i - L_i), \quad i = k+1,\ldots,k+d-1, i \text{ is } odd \end{cases}$$

$$t = \sum_{i=k+1}^{k+d-1} x_i$$

$$q(x_{k+1},\ldots x_{k+d}) = 100(1-\sin(\frac{\pi}{4}(\sum_{i=k+1}^{k+d-1} x_i - 2)))^2 + 50\sum_{i=k+2}^{k+d-1} i\cdot(2\sum_{j_1=k+2}^{k+d-1} x_{j_1} + \sum_{j_2=k+1}^{k+d-2} x_{j_2})^2$$
$$+50(k+d)(2x_{k+d} - x_1 - x_2)^2$$

$$h(x_{k+d+1},\cdots,x_D) = -20\exp(-0.2\frac{\sqrt{(\sum_{i=k+d+1}^{D}(x_i-0.2))^2}}{D-k-d}) - \exp(\frac{\sum_{i=k+d+1}^{D}\cos(2\pi(x_i-0.2))}{D-k-d}) + 20 + e$$

Its search space is:

$$x_i \in [0,1], i = 1,2,\ldots,k$$
$$x_i \in [-10,10], i = k+1,\ldots,k+d \text{ ,}$$
$$x_i \in [0,1], i = k+d+1,\ldots,D$$

Its PS1 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} (\frac{20x_1+4}{1+\cos(\frac{\pi}{2}\frac{i}{k+d-1})} - 10)/(1.5), j = odd \\ (\frac{20x_2-4}{1+\frac{i}{k+d-1}} - 10)/(1.5), j = even \end{cases}, j = k+1,\ldots,k+d$$

$$x_{k+d} = \frac{x_1+x_2}{2}, x_{k+d+1},\ldots,x_D = 0.2$$

Its PS2 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_j = \begin{cases} \left(\dfrac{20x_1 - 4}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+d-1})} - 10\right)/(1.5), j = odd \\ \left(\dfrac{20x_2 + 4}{1+\dfrac{i}{k+d-1}} - 10\right)/(1.5), j = even \end{cases}, j = k+1,...,k+d$$

$$x_{k+d} = \frac{x_1 + x_2}{2}, x_{k+d+1},..., x_D = 0.2$$

Its PF is:

$$f^2{}_1 + f_2^2 = 1,$$

where $0 \le f_1, f_2 \le 1$

NMMF14

$$k = 1, q_1 = 4, q_2 = 4, D \ge 10$$

$$\begin{cases} f_1 = x_1 \\ \dots \\ f_3 = g \cdot h \end{cases}$$

$$\text{for } l_1 \rightarrow \begin{cases} x_i^* = (1+\cos(\frac{\pi}{2}\cdot\frac{i}{k+q_1}))(2x_i - L_i) - \frac{x_1}{2}\cdot(U_i - L_i), \quad i = k+1,\dots,k+q_1, \ i \text{ is } \textit{even.} \\ x_i^* = (1+\frac{i}{k+q_1})(2x_i - L_i) - \frac{x_1}{2}\cdot(U_i - L_i), \quad i = k+1,\dots,k+q_1, \ i \text{ is } \textit{odd} \end{cases}$$

$$\text{for } l_2 \rightarrow \begin{cases} x_i^* = (1+\frac{i}{k+q_1+q_2})(2x_i - L_i) - \frac{x_{k+1}-L_{k+1}}{U_{k+1}-L_{k+1}}\cdot(U_i - L_i), \quad i = k+q_1+1,\dots,k+q_1+q_2, \ i \text{ is } \textit{even} \\ x_i^* = (1+\cos(\frac{\pi}{2}\cdot\frac{i}{k+q_1+q_2}))(2x_i - L_i) - \frac{x_{k+1}-L_{k+1}}{U_{k+1}-L_{k+1}}\cdot(U_i - L_i), \quad i = k+q_1+1,\dots,k+q_1+q_2, \ i \text{ is } \textit{odd.} \end{cases}$$

$$y_{1,i} = 50\cdot\sin\left(\sqrt{100(x_{i+1}-x_i)^2+(x_i-x_{i-1})^2}\right)^2 + 100r$$

$$y_{2,i} = 1+((x_{i+1}-x_i)^2 - 2(x_{i+1}-x_i)(x_i - x_{i-1}) + (x_i - x_{i-1})^2)^2$$

$$r = \begin{cases} 25(x-6)^2, x > 5 \\ \frac{24}{25}x^2 + 1, -5 \le x \le 5 \\ \frac{25}{4}(x+7)^2, x < -5 \end{cases}$$

$$g(x_{k+1},\dots x_{k+q_1}) = \sum_{i=2}^{D}(\frac{y_{1,i}}{y_{2,i}})$$

$$g(x_{k+q_1+1},\dots x_{k+q_1+q_2}) = \sum_{i=k+q_1+1}^{k+q_1+q_2-1}((|x_i| - \frac{k+i}{k+q_1+q_2})^2 + 100(x_{i+1} - \frac{k+i+1}{k+i}x_i)^2)^{1/4}\cdot(\sin^2(50((|x_i| - \frac{k+i}{k+q_1+q_2})^2$$
$$+10(x_{i+1} - \frac{k+i+1}{k+i}x_i)^2)^{1/10}) + 100)$$

$$g(x_{k+q_1+q_2+1},\cdots,x_D) = -20\exp(-0.2\frac{\sqrt{(\sum_{i=k+d+1}^{D}(x_i - 0.2))^2}}{D-k-d}) - \exp(\frac{\sum_{i=k+d+1}^{D}\cos(2\pi(x_i - 0.2))}{D-k-d}) + 20 + e$$

$$h = 1 - (\frac{x_1}{g})^2 - (\frac{x_1}{g})\cdot\sin(8\pi x_1)$$

Its search space is:

$$x_i \in [0,1], i = 1,2,\dots,k$$
$$x_i \in [-10,10], i = k+1,\dots,D\text{'}$$

Its PS1 is:

$$x_1 \in [0,1],,$$

$$x_j=\begin{cases}\left(\dfrac{20x_1+6}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1})}-10\right)/(2), j=odd\\ \left(\dfrac{20x_1+6}{1+\frac{i}{k+q_1}}-10\right)/(2), j=even\end{cases}, j=k+1,...,k+q_1$$

$$x_j=\begin{cases}\left(\dfrac{\frac{i}{k+q_1+q_2}+x_{k+1}+10}{1+\frac{i}{k+q_1+q_2}}-10\right)/(2), j=odd\\ \left(\dfrac{\frac{i}{k+q_1+q_2}+x_{k+1}+10}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})}-10\right)/(2), j=even\end{cases}, j=k+q_1+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D=0.2$$

Its PS2 is:

$x_1\in[0,1],$,

$$x_j=\begin{cases}\left(\dfrac{20x_1-7}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1})}-10\right)/(2), j=odd\\ \left(\dfrac{20x_1-7}{1+\frac{i}{k+q_1}}-10\right)/(2), j=even\end{cases}, j=k+1,...,k+q_1$$

$$x_j=\begin{cases}\left(\dfrac{\frac{i}{k+q_1+q_2}+x_{k+1}+10}{1+\frac{i}{k+q_1+q_2}}-10\right)/(2), j=odd\\ \left(\dfrac{\frac{i}{k+q_1+q_2}+x_{k+1}+10}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})}-10\right)/(2), j=even\end{cases}, j=k+q_1+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D=0.2$$

Its PS3 is:

$x_1\in[0,1],$,

$$x_j=\begin{cases}\left(\dfrac{20x_1-7}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1})}-10\right)/(2), j=odd\\ \left(\dfrac{20x_1-7}{1+\frac{i}{k+q_1}}-10\right)/(2), j=even\end{cases}, j=k+1,...,k+q_1$$

$$x_j=\begin{cases}\left(\dfrac{-\frac{i}{k+q_1+q_2}+x_{k+1}+10}{1+\frac{i}{k+q_1+q_2}}-10\right)/(2), j=odd\\ \left(\dfrac{-\frac{i}{k+q_1+q_2}+x_{k+1}+10}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})}-10\right)/(2), j=even\end{cases}, j=k+q_1+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D=0.2$$

Its PS4 is:

$x_1 \in [0,1]$, ,

$$x_j = \begin{cases} (\dfrac{20x_1+6}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1})} - 10)/(2), j = odd \\ (\dfrac{20x_1+6}{1+\dfrac{i}{k+q_1}} - 10)/(2), j = even \end{cases}, j = k+1,...,k+q_1$$

$$x_j = \begin{cases} (\dfrac{-\dfrac{i}{k+q_1+q_2} + x_{k+1} + 10}{1+\dfrac{i}{k+q_1+q_2}} - 10)/(2), j = odd \\ (\dfrac{-\dfrac{i}{k+q_1+q_2} + x_{k+1} + 10}{1+\cos(\dfrac{\pi}{2}\dfrac{i}{k+q_1+q_2})} - 10)/(2), j = even \end{cases}, j = k+q_1+1,...,k+q_1+q_2$$

$x_{k+q_1+q_2+1},...,x_D = 0.2$

Its PF is:

$$, \quad f_2 = 1 - f_1^2 - f_1 \cdot \sin(8\pi \cdot f_1)$$

where $0 \le f_1, f_2 \le 2$

NMMF15

$$k=3, q_1=6, q_2=12, D\geq 30$$

$$\begin{cases} f_1 = g_1(\theta)\left[1+W_1\cdot p(x_1,\cdots,x_k)^2+W_2\cdot q(x_{k+1},\cdots,x_{k+d})^2+W_3\cdot h(x_{k+d+1},\cdots,x_D)^2\right] \\ f_2 = g_2(\theta)\left[1+W_1\cdot p(x_1,\cdots,x_k)^2+W_2\cdot q(x_{k+1},\cdots,x_{k+d})^2+W_3\cdot h(x_{k+d+1},\cdots,x_D)^2\right] \end{cases}$$

$$\begin{cases} g_1(\theta)=\cos(\frac{\pi}{2}\theta) \\ g_2(\theta)=\sin(\frac{\pi}{2}\theta)\cos(\frac{\pi}{2}\theta) \end{cases}$$

$$\theta_1=\frac{2}{\pi}\arctan\left(\frac{\sqrt{{x_2}^2}}{x_1}\right)$$

$$p(x_1,\cdots,x_k)=1-\sum_{i=1}^{k}x_i^2$$

$$x^*=(1+\frac{i}{k+q_1})(2x_i-L\,)-\theta\cdot(U\;-L_i),\quad i=k+1,\ldots,k+q_1$$

$$\text{for } l_1\rightarrow\begin{cases} x_i^*=(1+\cos(\frac{\pi}{2}\cdot\frac{i}{k+q_1+q_2}))(2x_i-L_i)-\frac{x_{k+1}-L_{k+1}}{U_{k+1}-L_{k+1}}\cdot(U_i-L_i),\quad i=k+q_1+1,\ldots,k+q_1+q_2,\ i \text{ is } \textit{even.} \\ x_i^*=(1+\frac{i}{k+q_1+q_2})(2x_i-L_i)-\frac{x_{k+1}-L_{k+1}}{U_{k+1}-L_{k+1}}\cdot(U_i-L_i),\quad i=k+q_1+1,\ldots,k+q_1+q_2,\ i \text{ is } \textit{odd} \end{cases}$$

$$q(x_{k+1},\ldots,x_{k+q_1})=100\cdot(1-\sin(\frac{2}{8}\pi\cdot(t_1-2)))^2+50\cdot\sum_{i=k+2}^{k+q_1}i(0.7t_i-t_{i-1})^2, t_i=\sum_{j=k+1}^{i}x_j,\quad i=k+1,\ldots,k+q_1$$

$$q(x_{k+q_1+1},\ldots,x_{k+q_1+q_2})=1+\sum_{i=2}^{D}(\frac{y_{1,i}}{y_{2,i}})$$

$$y_{1,i}=50\cdot\sin\left(\sqrt{100(x_{i+1}-x_i)^2+(x_i-x_{i-1})^2}\right)^2+100r$$

$$y_{2,i}=1+((x_{i+1}-x_i)^2-2(x_{i+1}-x_i)(x_i-x_{i-1})+(x_i-x_{i-1})^2)^2$$

$$r=\begin{cases} 25(x-6)^2, x>5 \\ \left|0.508333333333333x^3-1.641666666666664x^2+2.5\right|, 0\leq x\leq 5 \\ 1.5(x+1)^2+1, -5<x<0 \\ \frac{25}{4}(x+7)^2, x\leq -5 \end{cases}$$

$$h(x_{k+q_1+q_2}+1,\ldots x_D)=20\exp(-0.2\frac{\sqrt{(\sum_{i=k+q_1+q_2+1}^{D}(x_i-0.2))^2}}{D-k-q_1-q_2})-\exp(\frac{\sum_{i=k+q_1+q_2+1}^{D}\cos(2\pi(x_i-0.2))}{D-k-q_1-q_2})+20+e$$

Its search space is:

$x_i \in [0,1], i = 1,2,...,k$

$x_i \in [-10,10], i = k+1,...,k+q_1+q_2$,

$x_i \in [0,10], i = k+q_1+q+1,...,D$

Its PS1 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_{k+1} = 4; x_{k+2} = (\frac{1-0.7}{0.7})^4$$

$$x_{k+i} = \frac{1}{0.7} x_{k+i-1}, i = 3,...,q_1$$

$$x_{k+i} = (\frac{20\theta + x_{k+i}}{1+\frac{i}{k+q_1}} - 10)/(2)$$

$$x_j = \begin{cases} (\frac{x_{k+1}+16}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})} - 10)/2, j = odd \\ (\frac{x_{k+1}+16}{1+\frac{i}{k+q_1+q_2}} - 10)/2, j = even \end{cases}, j = k+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D = 0.2$$

Its PS2 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_{k+1} = -4; x_{k+2} = (\frac{1-0.7}{0.7}) x_{k+1}$$

$$x_{k+i} = \frac{1}{0.7} x_{k+i-1}, i = 3,...,q_1$$

$$x_{k+i} = (\frac{20\theta + x_{k+i}}{1+\frac{i}{k+q_1}} - 10)/(2)$$

$$x_j = \begin{cases} (\frac{x_{k+1}+16}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})} - 10)/2, j = odd \\ (\frac{x_{k+1}+16}{1+\frac{i}{k+q_1+q_2}} - 10)/2, j = even \end{cases}, j = k+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D = 0.2$$

Its PS3 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_{k+1} = 4; x_{k+2} = (\frac{1-0.7}{0.7})x_{k+1}$$

$$x_{k+i} = \frac{1}{0.7}x_{k+i-1}, i = 3,...,q_1$$

$$x_{k+i} = (\frac{20\theta + x_{k+i}}{1+\frac{i}{k+q_1}} - 10)/(2)$$

$$x_j = \begin{cases} (\frac{x_{k+1}+3}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})} - 10)/2, j = odd \\ (\frac{x_{k+1}+3}{1+\frac{i}{k+q_1+q_2}} - 10)/2, j = even \end{cases}, j = k+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D = 0.2$$

Its PS4 is:

$$\sum_{i=1}^{k} x_i^2 = 1,$$

$$x_{k+1} = -4; x_{k+2} = (\frac{1-0.7}{0.7})x_{k+1}$$

$$x_{k+i} = \frac{1}{0.7}x_{k+i-1}, i = 3,...,q_1$$

$$x_{k+i} = (\frac{20\theta + x_{k+i}}{1+\frac{i}{k+q_1}} - 10)/(2)$$

$$x_j = \begin{cases} (\frac{x_{k+1}+3}{1+\cos(\frac{\pi}{2}\frac{i}{k+q_1+q_2})} - 10)/2, j = odd \\ (\frac{x_{k+1}+3}{1+\frac{i}{k+q_1+q_2}} - 10)/2, j = even \end{cases}, j = k+1,...,k+q_1+q_2$$

$$x_{k+q_1+q_2+1},...,x_D = 0.2$$

Its PF is:

,

$$f_1 + f_2 = 1$$

where $0 \le f_1, f_2 \le 1$