Supplementary File of “Knowledge Classification-assisted Evolutionary Multitasking for Two-task Multiobjective Optimization Problems”

This document contains the abbreviation and notations used in this work. It also provides details of test suites and experimental results (i.e., Fig. S1- S5 and Tables SⅠ- SⅦ) to complete Section Ⅳ.

**S.Ⅰ. Abbreviation**

The abbreviation and descriptions used in this paper are shown in this section.

|  |  |
| --- | --- |
| DE | differential evolution |
| EA | evolutionary algorithm |
| EMT | evolutionary multitasking |
| EMT-PD | evolutionary multitasking optimization algorithm based on population distribution |
| IGD | inverted generational distance |
| JDA | joint distribution adaptation |
| KNN | *k*-nearest neighbor |
| KC-EMT | knowledge classification-assisted evolutionary multitasking |
| KC-MOMFEA | knowledge classification-assisted multiobjective multifactorial evolutionary algorithm |
| KC-MOMFSA | knowledge Classification-assisted multiobjective multifactorial evolutionary algorithm based on subspace alignment |
| KC-MOMFDE | knowledge classification-assisted multiobjective multifactorial differential evolution |
| KC-SADE | knowledge classification-assisted subspace alignment and self-adaptive differential evolution |
| KC-TMO | knowledge classification-assisted two-stage assortative mating based multifactorial evolutionary algorithm for multi-objective optimization problems |
| MFEA | multifactorial evolutionary algorithm |
| MOEA | multiobjective evolutionary algorithm |
| MOEA/D | multiobjective evolutionary algorithm based on decomposition |
| MO-MFEA | multiobjective multifactorial evolutionary algorithm |
| MO-MFSA | multiobjective multifactorial evolutionary algorithm based on subspace alignment |
| MO-MFDE | multiobjective multifactorial differential evolution |
| ME-SADE | multiobjective evolutionary multitask optimization algorithm based on subspace alignment and self-adaptive differential evolution |
| MMD | maximum mean discrepancy |
| MOP | multiobjective optimization problem |
| MOPSO | multiobjective particle swarm optimization |
| MTMSO | multitasking multi-swarm optimization |
| MTO | multitask optimization |
| MTOP | multitask optimization problem |
| NSGA-II | non-dominated sorting genetic algorithm |
| PCA | principal component analysis |
| SA | subspace alignment |
| SPEA2 | strength pareto evolutionary algorithm |
| TMO-MFEA | two-stage assortative mating based multifactorial evolutionary algorithm for multi-objective optimization problems |
| VRP | vehicle routing problem |

## S.Ⅱ. Notations

The notations and descriptions used in this paper are shown in this section.

|  |  |
| --- | --- |
| **Notation** | **Description** |
| *Tj* | the *j*th task |
|  | the objective function |
|  | the number of elements in the objective function vector |
|  | the search space |
|  | decision variables |
|  | the Pareto-optimal set |
| *K* | the number of tasks |
|  | the high level group of target sub-population |
|  | the medium level group of target sub-population |
|  | the low level group of target sub-population |
|  | the number of fronts |
|  | the *i*th front |
|  | the performance rank value of the *i*th individual |
|  | the subspace dimension |
|  | a domain |
|  | feature space |
|  | a marginal probability distribution |
| ***𝕗*** | a classifier |
| **M**0 | an MMD matrix |
|  | examples that belong to the class of assistant sub-population |
|  | examples that belong to the class of target sub-population |
| **M***C* | an MMD matrix |
| **A** | transformation matrix in JDA |
| *C* | the number of classes |
|  | the unified search space |
|  | a label set |
|  | an expectation function |
|  | the problem dimension |
| *C* | the number of levels |
|  | the high level group of assistant sub-population |
|  | the medium level group of assistant sub-population |
|  | the low level group of assistant sub-population |
|  | the low level knowledge transfer archive |
|  | the medium level knowledge transfer archive |
|  | the initial population |
|  | the initial population size |
|  | target sub-population |
|  | assistant sub-population |
|  | subspace of target sub-population |
|  | subspace of assistant sub-population |
|  | a transformation matrix |
|  | the optimal transformation matrix |
|  | a mutually exclusive integers |
|  | a mutually exclusive integers |
|  | a mutually exclusive integers |
|  | the mutant vector |
|  | the trial vector |
|  | a scaling factor |
|  | a scaling factor |
|  | the mapped candidate of |
|  | the number of iterations for JDA |
|  | the number of the projected dimension used in JDA |
|  | candidate solution that has the best fitness value found at the generation |
|  | the number of trial vector generation strategies |
|  | the times that the *i*th trial vector strategy is employed |
|  | the probability of using the *i*th trial vector strategy |
|  | the reduction ratio |

## S.Ⅲ. Description of The Test Suites Used in Experiments

Tables SI-SII describe the multitask optimization problems contained in test suites 1 and 2, respectively.

TABLE SⅠ

Properties of Multiobjective Multitasking

Optimization Problems on Test Suite 1

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Task  No. | Pareto Set | Pareto Front | Inter-task Similarity |
| CIHS | *T*1 |  |  | 0.97 |
| *T*2 |  |  |
| CIMS | *T*1 |  |  | 0.52 |
| *T*2 |  |  |
| CILS | *T*1 |  |  | 0.07 |
| *T*2 |  |  |
| PIHS | *T*1 |  |  | 0.99 |
| *T*2 |  |  |
| PIMS | *T*1 |  |  | 0.55 |
| *T*2 |  |  |
| PILS | *T*1 |  |  | 0.002 |
| *T*2 |  |  |
| NIHS | *T*1 |  |  | 0.94 |
| *T*2 |  |  |
| NIMS | *T*1 |  |  | 0.51 |
| *T*2 |  |  |
| NILS | *T*1 |  |  | 0.001 |
| *T*2 |  |  |

TABLE SⅠI

Properties of Multiobjective Complex

Optimization Problems on Test Suite 2

|  |  |  |  |
| --- | --- | --- | --- |
|  | Task  No. | LZ09 Functions | Pareto Set |
| CPLX1 | *T*1 | *F*1 |  |
| *T*2 | *F*2 |  |
| CPLX2 | *T*1 | *F*1 |  |
| *T*2 | *F*7 |  |
| CPLX3 | *T*1 | *F*2 |  |
| *T*2 | *F*4 |  |
| CPLX4 | *T*1 | *F*2 |  |
| *T*2 | *F*9 |  |
| CPLX5 | *T*1 | *F*3 |  |
| *T*2 | *F*6 |  |
| CPLX6 | *T*1 | *F*3 |  |
| *T*2 | *F*9 |  |
| CPLX7 | *T*1 | *F*4 |  |
| *T*2 | *F*5 |  |
| CPLX8 | *T*1 | *F*5 |  |
| *T*2 | *F*7 |  |
| CPLX9 | *T*1 | *F*6 |  |
| *T*2 | *F*9 |  |
| CPLX10 | *T*1 | *F*7 |  |
| *T*2 | *F*8 |  |

## S.Ⅳ. Detailed Experimental Results

### *A. Parametric Analysis of the Subspace Dimension*

During the inter-task learning stage, the design of the subspace dimension is an important parameter that needs investigation. In this work, we set , where *d* is the original maximum dimension of unified search space. Other parameters are set in Section IV-A. We examine performance sensitivity with respect to this parameter, and it is set to 0.1, 0.3, 0.5, 0.7, and 0.9, respectively. Note that the value is decided by the dimension size of test problems. For example, the dimension size of most of the problems in test suite 1 is 50, and thus the minimum value of is 0.1. Tables SⅢ-SⅤ show the best achieved fitness values in 20 runs versus in KC-SADE on test suites 1-3, and the best one is shown in bold.

The computational results show that a small works well when tasks have high similarities or complete intersection decision space, such as the CIHS problem. This is because a small can transfer main components of each solution space, and thus improve the convergence speed in such cases. However, when handling tasks that have a low similarity or partial intersection, a small may fail to provide enough interaction knowledge for the target task, and even fall into local areas. According to the results in Tables SⅢ-SⅤ, a smaller such as 0.1 and 0.3 can encourage faster convergence speed, and improve the diversity of offspring solutions. Hence, we set to 0.1*d* to promise the stable performance of the proposed algorithms.

TABLE SⅢ

Computational Results of KC-SADE with Different Values on Test Suite 1

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pro. | **=0.1** | **=0.3** | **=0.5** | **=0.7** | **=0.9** |
| *P*1-*T*1  *P*1-*T*2 | 3.93e-03  **3.34e-04** | 3.96e-03  3.47e-04 | **3.92e-03**  3.45e-04 | **3.92e-03**  3.41e-04 | 3.95e-03  3.46e-04 |
| *P*2-*T*1  *P*2-*T*2 | **3.18e-04**  7.79e-04 | **3.18e-04**  7.77e-04 | 3.19e-04  **7.74e-04** | **3.18e-04**  7.77e-04 | 3.20e-04  7.81e-04 |
| *P*3-*T*1  *P*3-*T*2 | **7.66e-03**  **3.93e-04** | 7.69e-03  4.10e-04 | 7.69e-03  4.16e-04 | 7.69e-03  4.27e-04 | 7.69e-03  4.41e-04 |
| *P*4-*T*1  *P*4-*T*2 | 4.33e-04  4.41e-04 | 4.21e-04  **4.36e-04** | 4.29e-04  2.65e-02 | **4.11e-04**  4.44e-04 | 4.27e-04  2.65e-02 |
| *P*5-*T*1  *P*5-*T*2 | 7.79e-03  3.21e+01 | 8.48e-03  **2.14e+01** | **7.75e-03**  3.22e+01 | 9.05e-03  3.53e+01 | 9.02e-03  4.00e+01 |
| *P*6-*T*1  *P*6-*T*2 | 5.69e-03  7.85e-04 | 4.86e-03  **4.76e-04** | 6.15e-03  9.78e-02 | 7.07e-03  5.92e-02 | **4.85e-03**  9.72e-02 |
| *P*7-*T*1  *P*7-*T*2 | 4.12e+01  **4.24e-04** | 4.14e+01  4.32e-04 | **4.10e+01**  4.35e-04 | 4.14e+01  4.37e-04 | 4.15e+01  4.48e-04 |
| *P*8-*T*1  *P*8-*T*2 | **1.20e+01**  **3.72e-04** | 1.24e+01  6.91e-04 | 1.23e+01  1.12e-03 | 1.48e+02  1.59e-03 | 1.23e+01  7.98e-04 |
| *P*9-*T*1  *P*9-*T*2 | **5.86e-03**  **1.83e+01** | 9.01e-03  2.00e+01 | 1.30e-02  2.00e+01 | 1.05e-02  2.00e+01 | 8.75e-03  2.00e+01 |

TABLE SⅣ

Computational Results of KC-SADE with Different Values on Test Suite 2

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pro. | **=0.1** | **=0.3** | **=0.5** | **=0.7** | **=0.9** |
| *P*1-*T*1  *P*1-*T*2 | **5.30e-03**  **2.19e-02** | 5.35e-03  2.31e-02 | 5.39e-03  3.78e-02 | 5.42e-03  3.87e-02 | 5.39e-03  4.22e-02 |
| *P*2-*T*1  *P*2-*T*2 | 4.89e-03  2.14e-02 | **4.87e-03**  **2.00e-02** | 4.91e-03  2.11e-02 | 4.89e-03  2.07e-02 | 4.93e-03  2.15e-02 |
| *P*3-*T*1  *P*3-*T*2 | 6.27e-02  **2.56e-02** | 6.45e-02  2.65e-02 | 6.47e-02  2.64e-02 | **5.92e-02**  2.62e-02 | 5.94e-02  2.64e-02 |
| *P*4-*T*1  *P*4-*T*2 | 6.53e-02  6.36e-02 | 6.49e-02  6.91e-02 | **5.44e-02**  **5.91e-02** | 6.13e-02  6.73e-02 | 5.78e-02  6.06e-02 |
| *P*5-*T*1  *P*5-*T*2 | **3.03e-02**  **1.05e-01** | 3.31e-02  **1.05e-01** | 3.48e-02  **1.05e-01** | 3.50e-02  1.06e-01 | 3.54e-02  1.07e-01 |
| *P*6-*T*1  *P*6-*T*2 | **2.69e-02**  7.47e-02 | 3.19e-02  **7.16e-02** | 3.33e-02  7.77e-02 | 3.13e-02  7.93e-02 | 3.17e-02  7.59e-02 |
| *P*7-*T*1  *P*7-*T*2 | 2.60e-02  **2.00e-02** | **2.59e-02**  2.14e-02 | 2.62e-02  2.31e-02 | 2.66e-02  2.28e-02 | 2.64e-02  2.18e-02 |
| *P*8-*T*1  *P*8-*T*2 | **2.37e-02**  5.04e-02 | 2.43e-02  **3.44e-02** | 2.56e-02  6.27e-02 | 2.54e-02  8.51e-02 | 2.63e-02  4.09e-02 |
| *P*9-*T*1  *P*9-*T*2 | 1.08e-01  7.78e-02 | 1.08e-01  7.66e-02 | 1.09e-01  8.35e-02 | **1.06e-01**  7.89e-02 | **1.06e-01**  **6.90e-02** |
| *P*10-*T*1  *P*10-*T*2 | **6.79e-02**  **1.45e-01** | 1.20e-01  1.96e-01 | 8.15e-02  1.55e-01 | 7.11e-02  1.43e-01 | 9.43e-02  1.61e-01 |

TABLE SⅤ

Computational Results of KC-SADE with Different Values on Test Suite 3

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pro. | **=0.1** | **=0.3** | **=0.5** | **=0.7** | **=0.9** |
| *P*1-*T*1  *P*1-*T*2 | **3.18e+05**  **1.91e+05** | 3.59e+05  1.99e+05 | 3.76e+05  1.99e+05 | 3.84e+05  2.41e+05 | 4.82e+05  2.72e+05 |
| *P*2-*T*1  *P*2-*T*2 | **2.58e+01**  **2.59e+01** | 2.66e+01  2.63e+01 | 2.74e+01  2.75e+01 | 2.70e+01  2.67e+01 | 2.78e+01  2.74e+01 |
| *P*3-*T*1  *P*3-*T*2 | **2.73e+02**  **2.54e+02** | 3.72e+02  6.50e+02 | 2.88e+02  4.26e+02 | 3.12e+02  5.59e+02 | 5.02e+02  5.44e+02 |
| *P*4-*T*1  *P*4-*T*2 | 6.67e+00  7.14e+00 | **6.26e+00**  7.27e+00 | 7.00e+00  7.64e+00 | 6.39e+00  7.92e+00 | 7.57e+00  **6.84e+00** |
| *P*5-*T*1  *P*5-*T*2 | **5.90e+01**  **3.32e+01** | 6.78e+01  4.52e+01 | 7.04e+01  4.52e+01 | 7.35e+01  3.71e+01 | 6.75e+01  4.21e+01 |
| *P*6-*T*1  *P*6-*T*2 | 8.76e+01  8.23e+01 | **8.46e+01**  8.56e+01 | 9.78e+01  **8.16e+01** | 8.47e+01  9.00e+01 | 9.14e+01  8.34e+01 |
| *P*7-*T*1  *P*7-*T*2 | 3.40e+01  3.45e+01 | 3.70e+01  **3.05e+01** | **3.14e+01**  3.22e+01 | 3.35e+01  3.49e+01 | 3.67e+01  3.34e+01 |
| *P*8-*T*1  *P*8-*T*2 | 3.12e+03  **3.15e+03** | 3.55e+03  3.30e+03 | **2.37e+03**  3.38e+03 | 5.64e+03  3.76e+03 | 4.45e+03  4.08e+03 |
| *P*9-*T*1  *P*9-*T*2 | **5.80e+03**  **2.55e+03** | 6.40e+03  4.42e+03 | 6.17e+03  4.13e+03 | 6.00e+03  3.70e+03 | 6.29e+03  3.48e+03 |
| *P*10-*T*1  *P*10-*T*2 | 1.00e+01  4.42e+05 | 9.95e+00  **4.12e+05** | 1.07e+01  4.30e+05 | 1.14e+01  5.37e+05 | **9.60e+00**  4.32e+05 |

### *B. Detailed Convergence Traces*

Fig. S1 to Fig. S5 show detailed experimental results of IGD convergence curves of the proposed algorithm and its peers to complete Section Ⅳ-B. Each sub-picture shows the optimization process of *T*1 and *T*2. The Y-axis represents the IGD convergence curve and the X-axis denotes the number of generations.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (a) *P*1-*T*2 | (b) *P*2-*T*2 | (c) *P*3-*T*2 |
|  |  |  |
| (d) *P*4-*T*2 | (e) *P*5-*T*2 | (f) *P*6-*T*2 |
|  |  |  |
| (g) *P*7-*T*2 | (h) *P*8-*T*2 | (i) *P*9-*T*2 |

**Fig. S1.** IGD convergence traces of KC-SADE versus the NSGA-II, MO-MFEA, MO-MFEA-II, ME-SADE, TMO-MFEA, and EMT-PD on *T*2 of test suite 1.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (a) *P*1-*T*1 | (b) *P*2-*T*1 | (c) *P*3-*T*1 |
|  |  |  |
| (d) *P*4-*T*1 | (e) *P*5-*T*1 | (f) *P*6-*T*1 |
|  |  |  |
| (g) *P*7-*T*1 | (h) *P*8-*T*1 | (i) *P*9-*T*1 |
|  | | |
| (j) *P*10-*T*1 | | |

**Fig. S2.** IGD convergence traces of KC-SADE versus the NSGA-II, MO-MFEA, MO-MFEA-II, ME-SADE, TMO-MFEA, and EMT-PD on *T*1 of test suite 2.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (a) *P*1-*T*2 | (b) *P*2-*T*2 | (c) *P*3-*T*2 |
|  |  |  |
| (d) *P*4-*T*2 | (e) *P*5-*T*2 | (f) *P*6-*T*2 |
|  |  |  |
| (g) *P*7-*T*2 | (h) *P*8-*T*2 | (i) *P*9-*T*2 |
|  | | |
| (j) *P*10-*T*2 | | |

**Fig. S3.** IGD convergence traces of KC-SADE versus the NSGA-II, MO-MFEA, MO-MFEA-II, ME-SADE, TMO-MFEA, and EMT-PD on *T*2 of test suite 2.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (a) *P*1-*T*1 | (b) *P*2-*T*1 | (c) *P*3-*T*1 |
|  |  |  |
| (d) *P*4-*T*1 | (e) *P*5-*T*1 | (f) *P*6-*T*1 |
|  |  |  |
| (g) *P*7-*T*1 | (h) *P*8-*T*1 | (i) *P*9-*T*1 |
|  | | |
| (j) *P*10-*T*1 | | |

**Fig. S4.** IGD convergence traces of KC-SADE versus the NSGA-II, MO-MFEA, MO-MFEA-II, ME-SADE, TMO-MFEA, and EMT-PD on *T*1 of test suite 3.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| (a) *P*1-*T*2 | (b) *P*2-*T*2 | (c) *P*3-*T*2 |
|  |  |  |
| (d) *P*4-*T*2 | (e) *P*5-*T*2 | (f) *P*6-*T*2 |
|  |  |  |
| (g) *P*7-*T*2 | (h) *P*8-*T*2 | (i) *P*9-*T*2 |
|  | | |
| (j) *P*10-*T*2 | | |

**Fig. S5.** IGD convergence traces of KC-SADE versus the NSGA-II, MO-MFEA, MO-MFEA-II, ME-SADE, TMO-MFEA, and EMT-PD on *T*2 of test suite 3.

### *C. Comparison of Computational Time*

In order to represent the running time of different compared algorithms, Table SⅥ shows the computational time of KC-SADE and its peers when solving complex test suite 2. Experiments are conducted by using the computer with a 2.50 GHz IntelCorei5 processor and 16 GB RAM under window11. As shown in Table SⅥ, TMO-MFEA can improve the search speed because it divides decision variables into convergence- and diversity-related types, and thus improving the search efficiency of inter-task learning in the search space. For ME-SADE, it executes knowledge learning in lower dimensional space. Moreover, it uses the adaptive DE strategies to improve the search efficiency, and thus reduce the running time of ME-SADE.

In each generation, MO-MFEA-II needs to calculate probabilistic models and execute online parameter estimation to dynamically control the knowledge exchange among multiple tasks. Therefore, its computational time is longer. For our proposed algorithm, because of the additional knowledge classification strategy, its time cost is higher. However, this work focuses on the accuracy of knowledge transfer rather than the computational time. In our future work, we will attempt to design more effective knowledge classification methods to reduce running time of the proposed algorithm.

TABLE SⅥ

Comparison of the Average Computational Time (s) on Test Suite 2

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Problem |  | KC-SADE | NSGA-II | MO-MFEA | MO-MFEA-II | ME-SADE | TMO-MFEA | EMT-PD |
| *P*1 | mean  (std) | 1.34e+02  (1.90e+01) | 5.77e+02  (1.54e+01) | 5.98e+02  (1.72e+01) | 6.43e+02  (6.24e+00) | 8.64e+01  (4.98e+00) | 6.21e+01  (6.84e+00) | 6.92e+01  (2.64e+00) |
| *P*2 | mean  (std) | 1.45e+02  (8.71e+00) | 5.72e+02  (5.84e+00) | 5.87e+02  (3.19e+00) | 6.35e+02  (6.07e+00) | 8.55e+01  (1.35e+00) | 6.08e+01  (2.04e+00) | 6.94e+01  (9.24e-01) |
| *P*3 | mean  (std) | 1.50e+02  (8.84e+00) | 5.58e+02  (4.51e+00) | 5.61e+02  (4.77e+01) | 7.38e+02  (6.22e+00) | 8.86e+01  (1.49e+00) | 6.12e+01  (2.05e+00) | 8.17e+01  (3.08e+00) |
| *P*4 | mean  (std) | 1.46e+02  (3.24e+00) | 5.17e+02  (4.42e+01) | 5.37e+02  (3.64e+01) | 6.98e+02  (3.79e+01) | 8.86e+01  (1.62e+00) | 6.16e+01  (2.56e+00) | 8.12e+01  (2.47e+00) |
| *P*5 | mean  (std) | 1.58e+02  (1.55e+01) | 5.27e+02  (2.23e+01) | 6.07e+02  (5.00e+01) | 5.65e+02  (4.34e+00) | 9.14e+01  (3.09e+00) | 6.29e+01  (1.88e+00) | 8.34e+01  (1.77e+00) |
| *P*6 | mean  (std) | 1.46e+02  (2.67e+00) | 4.66e+02  (3.60e+00) | 5.98e+02  (6.18e+00) | 6.29e+02  (8.86e+00) | 8.93e+01  (2.57e+00) | 6.12e+01  (2.00e+00) | 8.20e+01  (2.10e+00) |
| *P*7 | mean  (std) | 1.45e+02  (2.94e+00) | 4.74e+02  (2.61e+01) | 5.76e+02  (9.94e+00) | 6.09e+02  (4.48e+00) | 8.85e+01  (1.58e+00) | 6.08e+01  (1.63e+00) | 7.79e+01  (2.59e+00) |
| *P*8 | mean  (std) | 1.48e+02  (1.27e+00) | 5.83e+02  (4.14e+00) | 5.51e+02  (4.79e+01) | 6.42e+02  (1.08e+01) | 9.32e+01  (8.24e+00) | 6.24e+01  (1.96e+00) | 7.46e+01  (8.34e+00) |
| *P*9 | mean  (std) | 1.50e+02  (3.18e+00) | 5.92e+02  (4.29e+01) | 5.66e+02  (1.81e+01) | 6.96e+02  (4.29e+01) | 9.16e+01  (2.23e+00) | 5.88e+01  (1.16e+01) | 8.50e+01  (1.02e+00) |
| *P*10 | mean  (std) | 1.24e+02  (6.58e+00) | 5.05e+02  (5.27e+00) | 5.60e+02  (3.68e+01) | 6.10e+02  (5.02e+01) | 8.77e+01  (1.39e+00) | 6.58e+01  (3.25e+00) | 7.26e+01  (8.89e-01) |

### *D. The Effect of Knowledge in Different Levels*

In this work, the population is divided into three level groups. To further investigate which layer of knowledge to transfer is more beneficial, this section investigates the effect of individuals in different levels. We use MO-MFDE as a baseline algorithm, and it generates offspring solutions via conventional differential evolution solvers. KC-SADE#H and KC-SADE#M are two variants of KC-SADE. The former uses individuals in the high-level group to provide knowledge for those in the low-level group. The latter uses individuals in the medium-level group to provide knowledge for those in the low-level group. Table SⅦ shows the computational results on test suite 3.

As shown in Table SⅦ, both KC-SADE#H and KC-SADE#M can enhance the solving ability in comparison with MO-MFDE. This is because knowledge extracted from high and medium levels can provide positive guide for low-level individuals. However, for KC-SADE, high and medium levels work together to provide the evolution experience for the whole population, and thus KC-SADE performs the best in Table SⅦ. When comparing KC-SADE#H to KC-SADE#M, we can find that KC-SADE#H possesses better performance. The high-level group usually contains individuals with better performance, and thus offers more useful knowledge to guide the solution evolution.

TABLE SⅦ

Experimental Results of The Effect of Knowledge in Different Levels

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Pro. |  | MO-MFDE | KC-SADE#H | KC-SADE#M | KC-SADE |
| *P*1-*T*1  *P*1-*T*2 | mean(std.)  mean(std.) | 2.39e+06(1.22e+06)  2.51e+06(1.37e+06) | 5.12e+05(3.83e+05)  3.10e+05(1.45e+05) | 6.36e+05(3.67e+05)  3.75e+05(1.92e+05) | **3.18e+05**(3.01e+05)  **1.91e+05**(1.25e+05) |
| *P*2-*T*1  *P*2-*T*2 | mean(std.)  mean(std.) | 3.23e+01(2.21e+00)  3.23e+01(2.34e+00) | 3.05e+01(2.53e+00)  3.03e+01(3.03e+00) | 2.97e+01(3.09e+00)  2.94e+01(3.10e+00) | **2.58e+01**(2.45e+00)  **2.59e+01**(2.29e+00) |
| *P*3-*T*1  *P*3-*T*2 | mean(std.)  mean(std.) | 6.12e+02(3.17e+02)  5.01e+02(2.17e+02) | 4.84e+02(2.76e+02)  4.96e+02(2.42e+02) | 5.34e+02(2.15e+02)  5.85e+02(3.16e+02) | **2.73e+02**(2.05e+02)  **2.54e+02**(2.25e+02) |
| *P*4-*T*1  *P*4-*T*2 | mean(std.)  mean(std.) | 1.40e+01(8.80e+00)  1.80e+01(9.59e+00) | **6.31e+00**(1.76e+00)  **7.10e+00**(2.81e+00) | 7.33e+00(2.37e+00)  8.37e+00(2.51e+00) | 6.67e+00(1.49e+00)  7.14e+00(2.67e+00) |
| *P*5-*T*1  *P*5-*T*2 | mean(std.)  mean(std.) | 1.28e+02(4.31e+01)  1.21e+02(5.43e+01) | 7.96e+01(4.85e+01)  5.37e+01(3.25e+01) | 8.50e+01(3.59e+01)  6.04e+01(4.24e+01) | **5.90e+01**(3.29e+01)  **3.32e+01**(3.25e+01) |
| *P*6-*T*1  *P*6-*T*2 | mean(std.)  mean(std.) | 1.01e+02(2.02e+01)  1.02e+02(2.27e+01) | 1.00e+02(1.90e+01)  **8.08e+01**(1.39e+01) | 9.76e+01(2.20e+01)  9.75e+01(2.00e+01) | **8.76e+01**(1.73e+01)  8.23e+01(1.98e+01) |
| *P*7-*T*1  *P*7-*T*2 | mean(std.)  mean(std.) | **2.20e+01**(4.89e+00)  **2.43e+01**(5.74e+00) | 3.49e+01(8.93e+00)  4.05e+01(9.40e+00) | 3.39e+01(1.06e+01)  4.43e+01(9.28e+00) | 3.40e+01(3.89e+00)  3.45e+01(9.48e+00) |
| *P*8-*T*1  *P*8-*T*2 | mean(std.)  mean(std.) | 4.84e+03(6.72e+03)  2.33e+04(1.24e+04) | 3.61e+03(2.03e+03)  5.11e+03(3.02e+03) | 5.28e+03(5.58e+03)  4.74e+03(2.11e+03) | **3.12e+03**(3.37e+03)  **3.15e+03**(1.47e+03) |
| *P*9-*T*1  *P*9-*T*2 | mean(std.)  mean(std.) | 8.61e+03(1.21e+03)  7.76e+03(6.21e+03) | 7.76e+03(1.93e+03)  4.70e+03(5.43e+03) | 8.20e+03(2.11e+03)  5.50e+03(4.97e+03) | **5.80e+03**(6.81e+02)  **2.55e+03**(3.56e+03) |
| *P*10-*T*1  *P*10-*T*2 | mean(std.)  mean(std.) | 1.62e+01(3.90e+00)  5.26e+06(3.03e+06) | 1.18e+01(3.85e+00)  **3.95e+05**(2.14e+05) | 1.09e+01(3.56e+00)  4.79e+05(2.31e+05) | **1.00e+01**(3.22e+00)  4.42e+05(2.31e+05) |