Letter

Supplementary Material of "Synchronous Membership Function Dependent Event-Triggered H_∞ Control of T-S Fuzzy Systems Under Network Communications"

Bo-Lin Xu, Chen Peng, *Senior Member, IEEE*, and Wen-Bo Xie, *Member, IEEE*

APPENDIX

Proof of Theorem 1:

Choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}V(t,x(t)) &= x^T(t)Px(t)+\eta_2^2\int_{i_kh}^{t}\dot{x}^T(s)W\dot{x}(s)ds\\&+\sum_{i=1}^{2}\int_{t-\eta_i}^{t-\eta_{i-1}}x^T(s)Q_ix(s)ds\\&+\sum_{i=1}^{2}(\eta_i-\eta_{i-1})\int_{t-\eta_i}^{t-\eta_{i-1}}\int_s^t\dot{x}^T(v)R_i\dot{x}(v)dvds\\&-\frac{\pi^2}{4}\int_{i_kh}^{t-\eta_1}[x(s)-x(i_kh)]^TW[x(s)-x(i_kh)]ds\end{aligned}\quad(16)$$

where $\eta_0=0$, the time derivative of $V(t,x(t))$ is

$$\begin{aligned}\dot{V}(t,x(t)) &= 2x^T(t)P\dot{x}(t)+x^T(t)Q_1x(t)\\&-x^T(t-\eta_1)Q_1x(t-\eta_1)+x^T(t-\eta_1)Q_2x(t-\eta_1)\\&-x^T(t-\eta_2)Q_2x(t-\eta_1)+\sum_{i=1}^{2}(\eta_i-\eta_{i-1})^2\dot{x}^T(t)R_i\dot{x}(t)\\&-\sum_{i=1}^{2}(\eta_i-\eta_{i-1})\int_{t-\eta_i}^{t-\eta_{i-1}}\dot{x}^T(v)R_i\dot{x}(v)dv\\&+\eta_2^2\dot{x}^T(t)W\dot{x}(t)+e^T(i_kh)\Phi e(i_kh)-e^T(i_kh)\Phi e(i_kh)\\&-\frac{\pi^2}{4}[x(t-\eta_1)-x(t-\eta(t))]^TW[x(t-\eta_1)-x(t-\eta(t))].\end{aligned}\quad(17)$$

For zero-initial state and define J as

$$\begin{aligned}J &= \int_0^{+\infty}y^T(t)y(t)dt-\gamma^2\int_0^{+\infty}\omega^T(t)\omega(t)dt\\&=\int_0^{+\infty}(y^T(t)y(t)-\gamma^2\omega^T(t)\omega(t)+\dot{V}(t,x(t)))dt\\&-V(+\infty,x(+\infty))\end{aligned}\quad(18)$$

Lemma 1 and Jensen inequality are adopted to relax the integral term in (17). Define $\varrho^T(t)=[x^T(t),x^T(t-\eta_1),x^T(t-\eta(t)),x^T(t-\eta_2),e^T(i_kh),\omega^T(t)]$, $X=P^{-1}$, $XQ_iX=\widehat{Q}_i$, $XR_iX=\widehat{R}_i$, $X\Phi X=\widehat{\Phi}$, $XUX=\widehat{U}$ and $Y_j=K_jP^{-1}$. Pre- and post-multiply (17) by matrix diag$\{X\ X\ X\ X\ X\ I\ R_1^{-1}\ R_2^{-1}\ W^{-1}\ I\}$ and its transpose, respectively. Lemma 2 is used to deal with terms $X\widehat{R}_i^{-1}X$ and $X\widehat{W}^{-1}X$. For $V(+\infty,x(+\infty))\geq 0$, using Schur complement, the sufficient condition for $J<0$ is

$$\sum_{i=1}^{l}\sum_{j=1}^{l}\varrho^T(t)\vartheta_i(\vartheta_j+\epsilon_j)\widehat{\Pi}_{ij}\varrho(t)<0\quad(19)$$

where $\widehat{\Pi}_{ij}$ is detailed in Theorem 1. For $S_{ij}>0$ and condition (12), setting $\vartheta_{ij}=\vartheta_i\vartheta_j$, one can have

$$\begin{aligned}\sum_{i=1}^{l}\sum_{j=1}^{l}\vartheta_i(\vartheta_j+\epsilon_j)\widehat{\Pi}_{ij} &= \sum_{i=1}^{l}\sum_{j=1}^{l}\vartheta_{ij}[\widehat{\Pi}_{ij}+\epsilon_j(\widehat{\Pi}_{ij}+S_{ij})-\epsilon_jS_{ij}]\\&\leq\sum_{i=1}^{l}\sum_{j=1}^{l}\vartheta_{ij}\Lambda_{ij}\end{aligned}\quad(20)$$

where Λ_{ij} is detailed in Theorem 1. Utilizing linear piecewise approximation method mentioned in Proposition 1, for $T_{ij}>0$ and condition (13), a sufficient condition for guaranteeing the negativeness of (20) is given as

$$\begin{aligned}\sum_{i=1}^{l}\sum_{j=1}^{l}\vartheta_{ij}\Lambda_{ij} &= \sum_{i=1}^{l}\sum_{j=1}^{l}(\hat{\vartheta}_{ij}+\Delta\vartheta_{ij})\Lambda_{ij}\\&=\sum_{i=1}^{l}\sum_{j=1}^{l}[\hat{\vartheta}_{ij}\Lambda_{ij}+\Delta\vartheta_{ij}(\Lambda_{ij}+T_{ij})-\Delta\vartheta_{ij}T_{ij}]\\&=\zeta_{\varpi p}\{\sum_{i=1}^{l}\sum_{j=1}^{l}[\hat{\vartheta}_{ij}(z_{i_1\ldots i_p\psi})\Lambda_{ij}+\Delta\vartheta_{ij}(z_{i_1\ldots i_p\psi})\\&\quad(\Lambda_{ij}+T_{ij})-\Delta\vartheta_{ij}(z_{i_1\ldots i_p\psi})T_{ij}]\}\\&\leq\zeta_{\varpi p}\{\sum_{i=1}^{l}\sum_{j=1}^{l}[\hat{\vartheta}_{ij}(z_{i_1\ldots i_p\psi})\Lambda_{ij}+\Delta\overline{\vartheta}_{ij}(z_{i_1\ldots i_p\psi})\\&\quad(\Lambda_{ij}+T_{ij})-\Delta\underline{\vartheta}_{ij}(z_{i_1\ldots i_p\psi})T_{ij}]\}\end{aligned}\quad(21)$$

where $\zeta_{\varpi p}=\sum_{\psi=1}^{\varpi}\sum_{i_1=1}^{2}\ldots\sum_{i_p=1}^{2}\prod_{r=1}^{p}v_{ri_r\psi}(z_r)$. Thus, condition (14) is obtained. Moreover, when using Lemma 1, pre-multiply and post-multiply $\begin{bmatrix}R_2 & *\\ U & R_2\end{bmatrix}>0$ by matrix diag$\{X\ \ X\}$ and its transpose, respectively. Condition (15) is obtained. ■